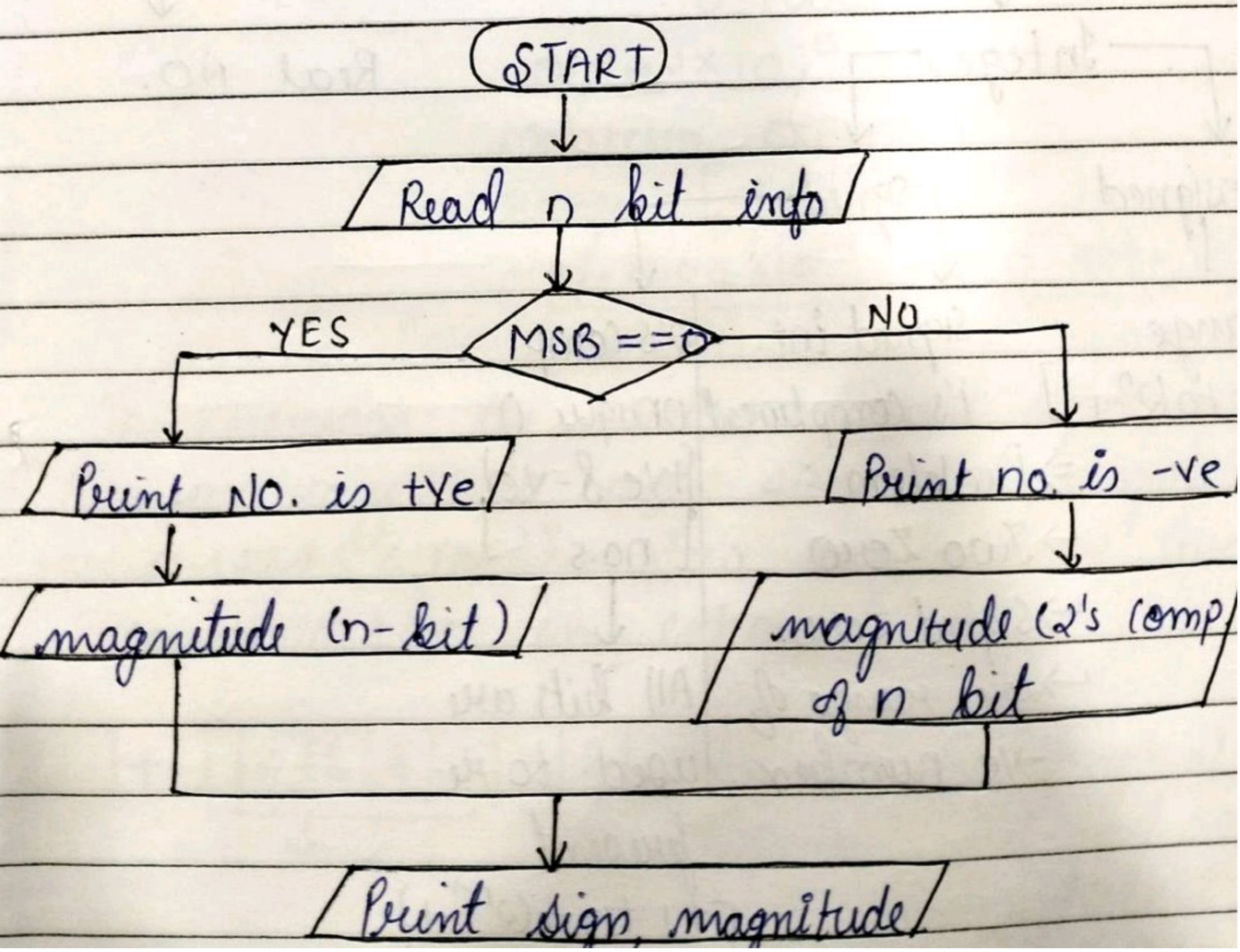
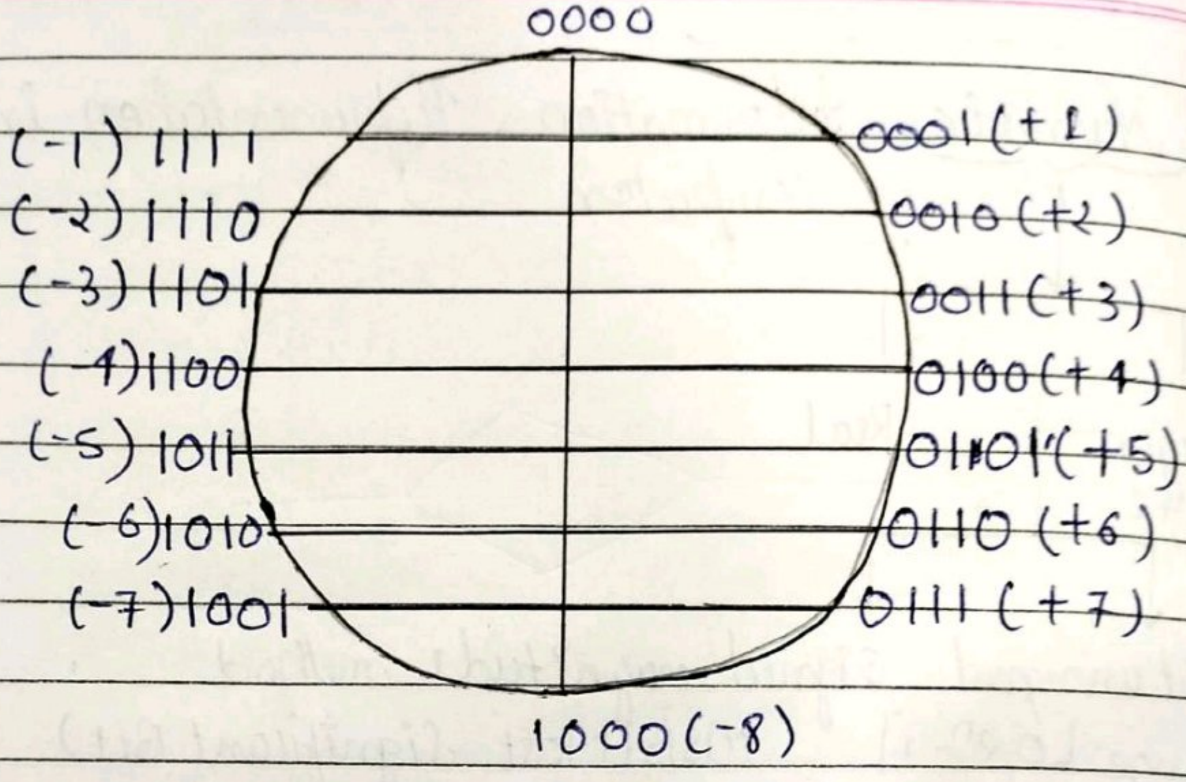
Numeric Information Representation in computer

Integer / Real

Integer: -2, +2

- n-bit unsigned
  - range: $[0, 2^n - 1]$
- Signed magnitude method
  - MSB (Most Significant Bit) → sign
- and 2's compliment
  - range: $[-2^{n-1}, +(2^{n-1} - 1)]$





05/10/2023

Numeric Information
- Integer
  - Unsigned
    - Range $[0 \text{ to } (2^n - 1)]$
  - Signed
    - Signed int.
      - 1's compliment
      - ⇒ Problem :
      - → Two Zero
      - ↳ Signed zero
      - ↳ Less range of -ve number
    - 2's comp.
      - unique 0
      - {+ve & -ve no.s}
      - ↓
      - All bits are used to represent
      - $[+1 \text{ to } +(2^{n-1} - 1)$
      - $-1 \text{ to } -(2^{n-1})$
- Real NO.

Real NO

fixed point ×

max +999.999

min −999.999

only 7 places

Normalization

$[N = M \times 10^E]$

convert in (0,1)

format :- 0. _ _ _

Non zero

eg:- −420.007

$\Rightarrow 0.420007 \times 10^{+3}$

representation :- −0.420007E+

converting into floating Pt

eg:- 0.00007

0.7000×10^{-1}

$\underbrace{0.402}_{\text{Mantessa}} \times \underbrace{10^{-5}}_{\text{exponent}}$

range :- $[-10^{-100}, 10^{+99}]$

$\underbrace{-0.1000 \times 10^{-99}}_{\text{min}}$, $\underbrace{+0.999 \times 10^{+99}}_{\text{max}}$

To increase the range without affecting the accuracy do not store the sign

$\Rightarrow 0.12345 \times 10^{-28}$ Now we'll store this so add +50 in exponent

+	• 1	2	3	4	5	2	2

$\underbrace{1\ 2\ 3\ 4\ 5}_{M}$ $\underbrace{2\ 2}_{E}$

to get the original exponent subtract 50 from the exponent that we have added +50

Sign magnitude / 1's 2's Complement

START

↓

Read n bit binary no.

↓

MSB = 0 ?

- YES → Sign = +
- NO → Sign = −

↓

mag = (n-1) bits ecluding MSB

↓

Print Sign, mag

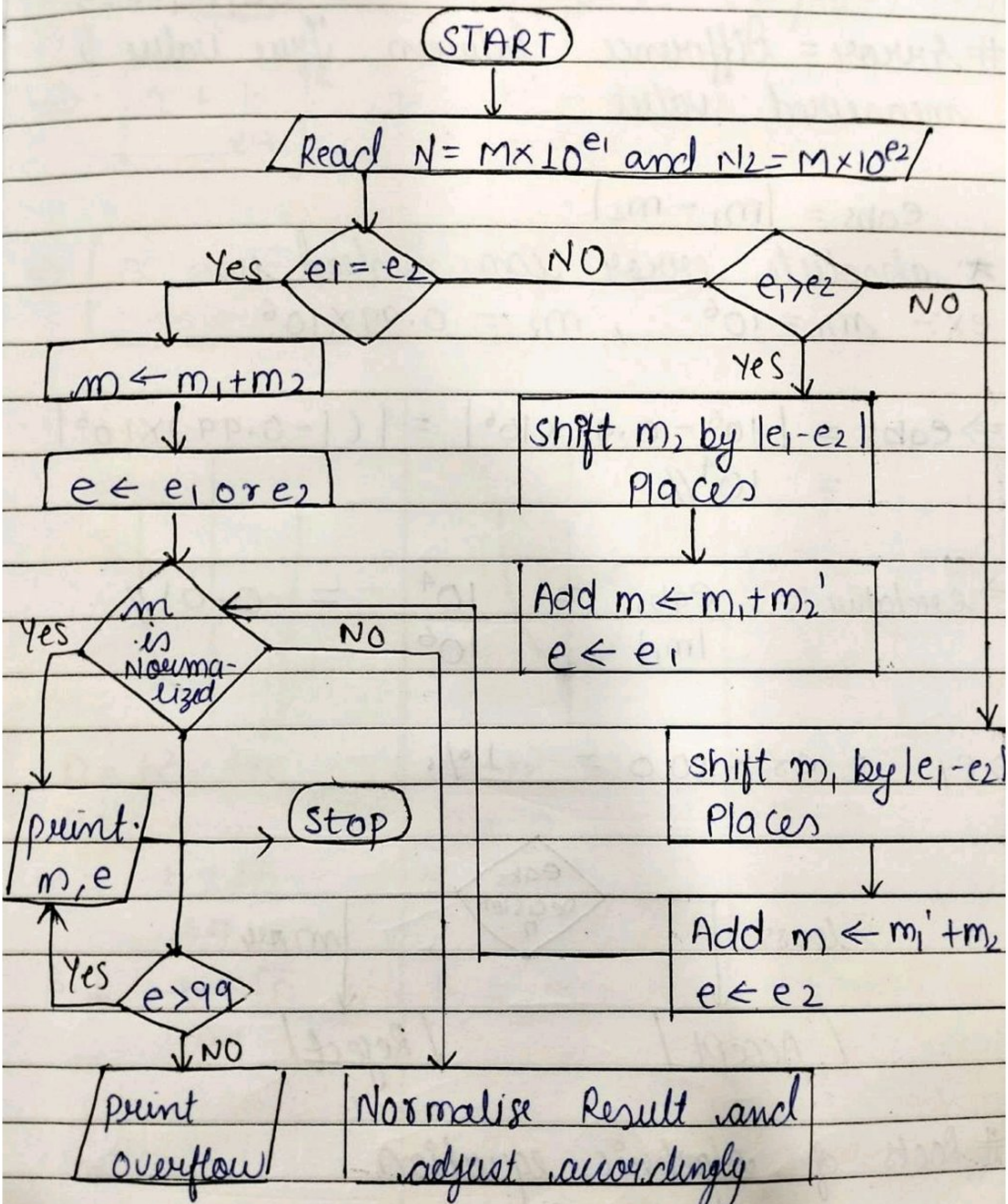
↓

STOP

Numeric Info

- Integer
  - Unsigned
  - Signed
    - × 1's compliment
    - ✓ 2's compliment
- Real
  - fixed Point → ×
  - floating Point → Normalization
    - Mantissa
    - Exponent

To perform addition, subtraction, multiplication & division on floating point



Substraction is same as addition

$\Rightarrow$ Multiplication $= N_1 \times N_2 = m \times 10^{e_1} \times m_2 \times 10^{e_2}$
$= (m_1 \times m_2)^{e_1+e_2}$

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Error = Difference between True value & measured value

$$e_{abs} = |m_1 - m_2|$$

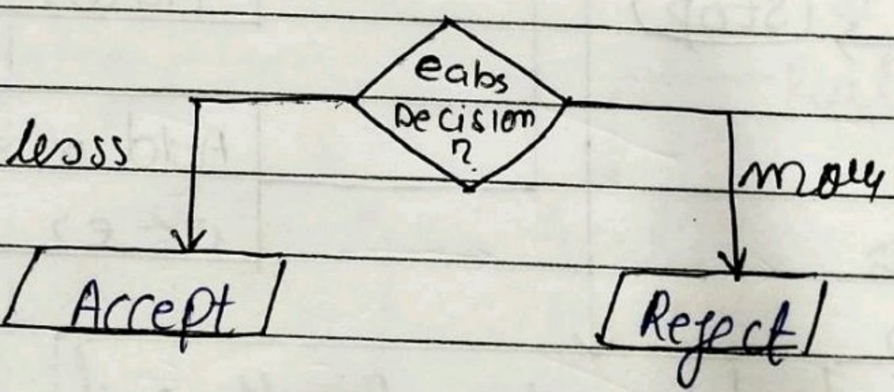
★ absolute error can mislead us

ex:- $m_1 = 10^6$, $m_2 = 0.99 \times 10^6$

$\Rightarrow e_{abs} = |10^6 - 0.99 \times 10^6| = |(1-0.99) \times 10^6|$
$= 10^4$ //

$\Rightarrow e_{relative} = \dfrac{e_{abs}}{|m_1|} = \dfrac{10^4}{10^6} = 0.01$ //

$\Rightarrow e_{\%} = e_r \times 100 = 1\%$



Roots of algebric equation

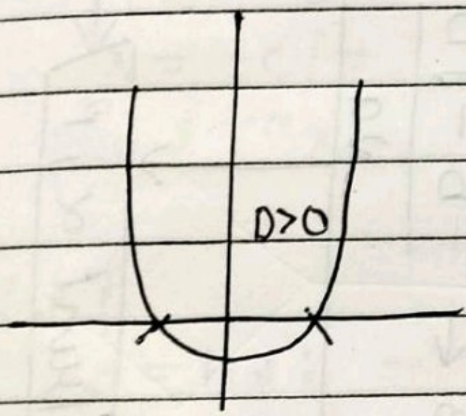
$\Rightarrow ax^2 + bx + c = 0$
$\Rightarrow ax^2 + bx = -c$
$\Rightarrow x^2 + bx = -c$

add $(b/2a)^2$ both side

$$\Rightarrow x^2 + (b/a)x + (b^2/4a^2) = (b^2/4a^2) - (c/a)$$

$$\Rightarrow \left[x + \frac{b}{2a}\right]^2 = \frac{b^2 - 4ac}{4a^2}$$

$$\Rightarrow \boxed{x = \frac{-b \pm \sqrt{4ac}}{2a}}$$

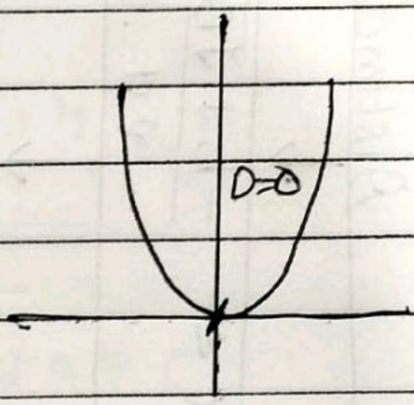


$D = b^2 - 4ac$

$D > 0$

$$\alpha = \frac{-b + \sqrt{D}}{2a}$$

$$\beta = \frac{-b - \sqrt{D}}{2a}$$

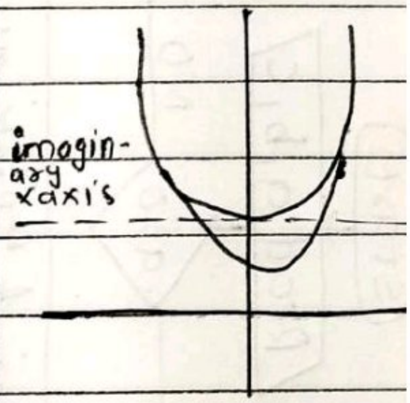


$D = b^2 - 4ac$

$D = 0$

$$\alpha = \frac{-b}{2a}$$

$\beta = \alpha$



$D = b^2 - 4ac$

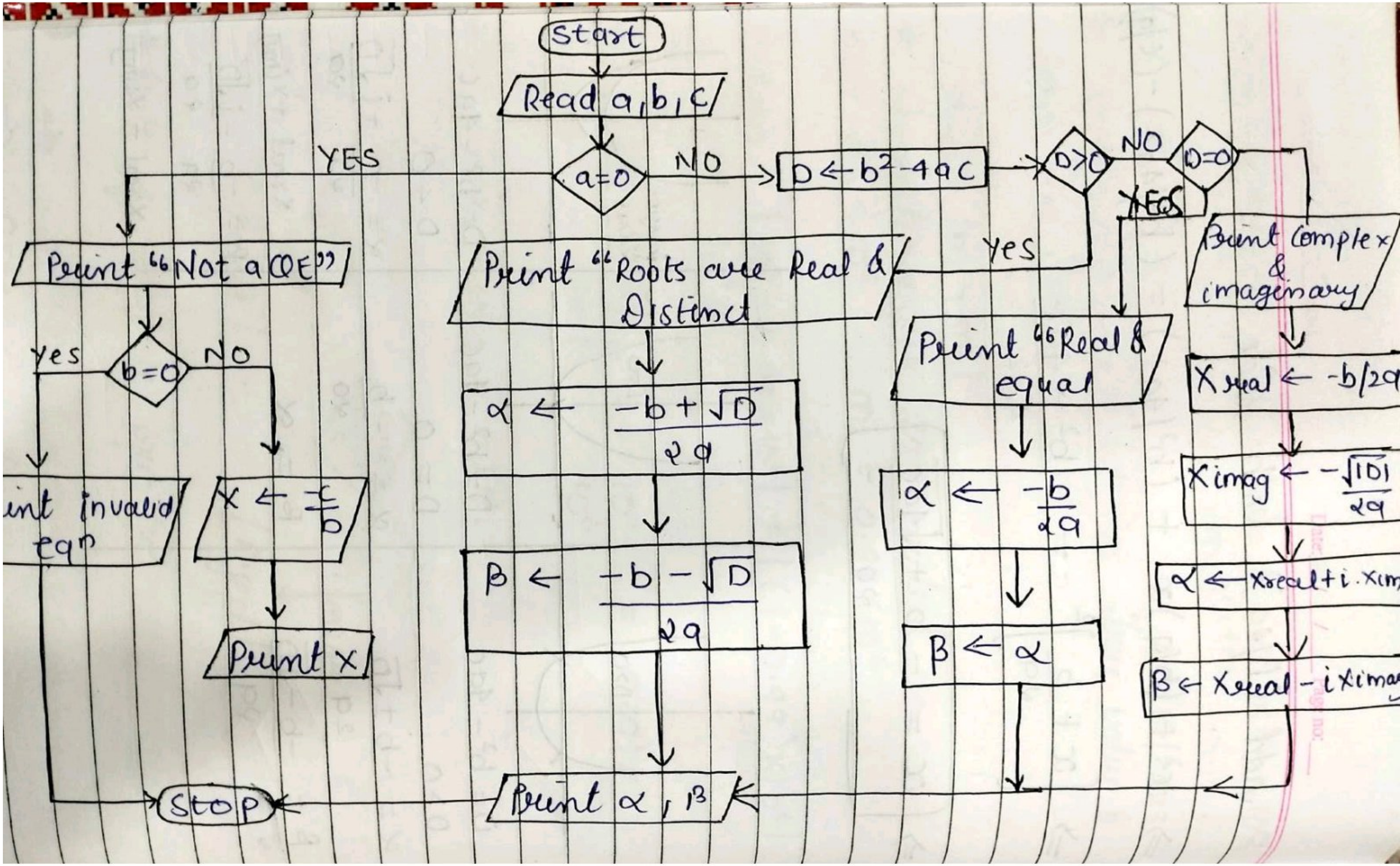
$D < 0$

$$\alpha = \frac{-b}{2a} + \frac{i\sqrt{D}}{2a}$$

$x_{real} + x_{imag}$

$$\beta = \frac{-b}{2a} - \frac{i\sqrt{D}}{2a}$$

$x_{real} - x_{imag}$


Start
Read a, b, c
a=0
YES
NO
D ← b²-4ac
D>0
NO
D=0
YES
yes
Print "Not a QE"
b=0
yes
NO
Print invalid eqn
x ← -c/b
Print x
Print "Roots are Real & Distinct
α ← (-b + √D)/2a
β ← (-b - √D)/2a
Print "Real & equal
α ← -b/2a
β ← α
Print complex & imaginary
X real ← -b/2a
X imag ← -√|D|/2a
α ← Xreal + i.Ximag
β ← Xreal - iXimag
Print α, β
Stop

```
# include< math.h>
void main ( )
{ float a, b, c, d, xreal, ximag, alpha, beta
clrscr( );
printf("\n enter coeff. of QE a,b and c\n");
scanf("%f %f %f", &a, &b, &c);
if(a==0)
{ printf ("Not a Q.E\n");
  if(b!=0)
   x=-c/b;
  print ("x = %f", x);
}
else {
   d= b*b - 4*a*c;
   if (d>0 )
  { printf ("Root are real & distinct ");
   alpha = [(-b)+sqrt(d)]/(2*a)];
   beta  = ((-b) -sqrt(d))/(2*a);
  }
   if (d==0 )
  { printf ("Roots are real & imaginary");
   alpha = -b/(2*a);
   beta  = alpha;
  }
 printf (" Roots of QE => x1 = %f and x2=
            %f", alpha, beta );
```

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```
else
{printf("roots are complex & imaginary\n");
 xreal = ((-b)/(2*a));
 ximag = (sqrt(d)/(2*a));
 printf("first complex root = %f + i%f",
            xreal, ximag);
 printf("Second complex root = %f - i%f",
            xreal, x ximag);
}
 getch();
}
```

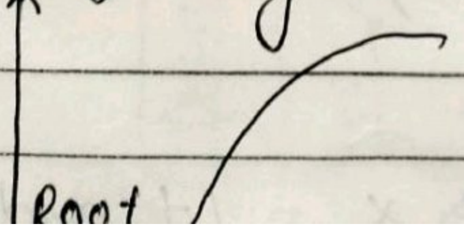
07/10/2023

Algorithm :- Algorithm is a finit set of instruction if followed to accomplish a given task.

⇒ Properties of Algorithm :-

↳ Finitness
↳ Definiteness
↳ Effectiveness
↳ task input $(n \geq 0)$
↳ give output $(n \geq 1)$

⇒ Root Finding Methods :-



```
else
{ printf ("roots are complex & imaginary\n");
  xreal = ((-b)/(2*a));
  ximag = (sqrt(d)/(2*a));
  printf("first complex root = %f + i%f",
          xreal, ximag);
  printf("Second complex root = %f - i%f",
          xreal, x ximag);
}
getch();
}
```

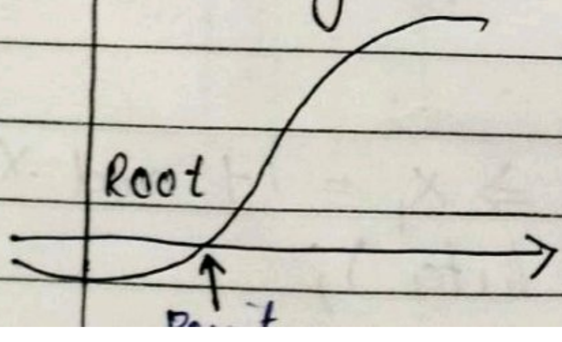
07/10/2023

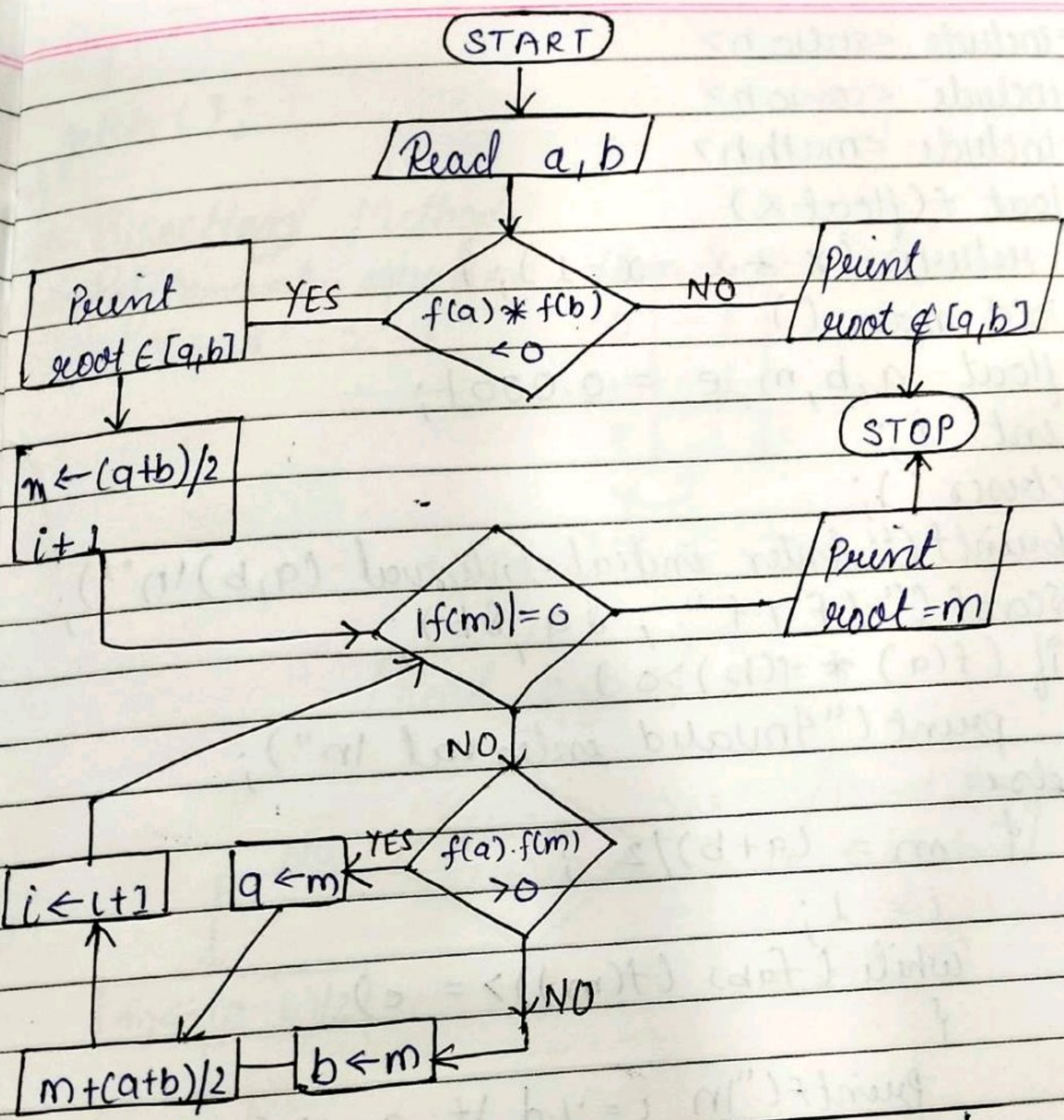
Algorithm :- Algorithm is a finit set of instruction of followed to accomplish a given task.

⇒ Properties of Algorithm :-

↪ Finitness
↪ Definiteness
↪ Effectiveness
↪ take input (n ≥ 0)
↪ give output (n ≥ 1)

⇒ Root Finding Methods :-





ex: $f(x) = x^2 - x - 1$

$\Rightarrow x = 0$ $\quad f(0) = -ve$

$\Rightarrow x = 1$ $\quad f(1) = -ve$

$\Rightarrow x = 2$ $\quad f(2) = +ve$

$f(1) * f(2) < 0$

$\downarrow$

root ← [1,2]

i	a	b	$m = \frac{a+b}{2}$	f(m)
1	1.000	2.000	1.5000	=0.25
2	1.5000	2.000	1.75	$(1.75)^2 - 1.75$
3	1.500	1.750	1.6250	
4				

```
#include <stdio.h>
#include <conio.h>
#include <math.h>
float f(float x)
{ return (x * x - x - 1); }
void main ()
{ float a, b, m, e = 0.0001;
 int i;
 clrscr ();
 printf("Enter initial interval (a,b)\n");
 scanf("%f %f", &a, &b);
 if (f(a) * f(b) > 0 )
    print("Invalid interval \n");
 else
  { m = (a+b)/2 ;
    i = 1;
   while (fabs (f(m)) >= e)
   {
     printf("\n i = %d \t a = %f \t
              b = %f \t m = %f \t
              f(m) = %f \n" i, a, b, m, f(m));

     if(f(a) * f(m) > 0 )
        a = m;
     else
        b = m;
        m = (a+b)/2 ;
```

}
getch();
}

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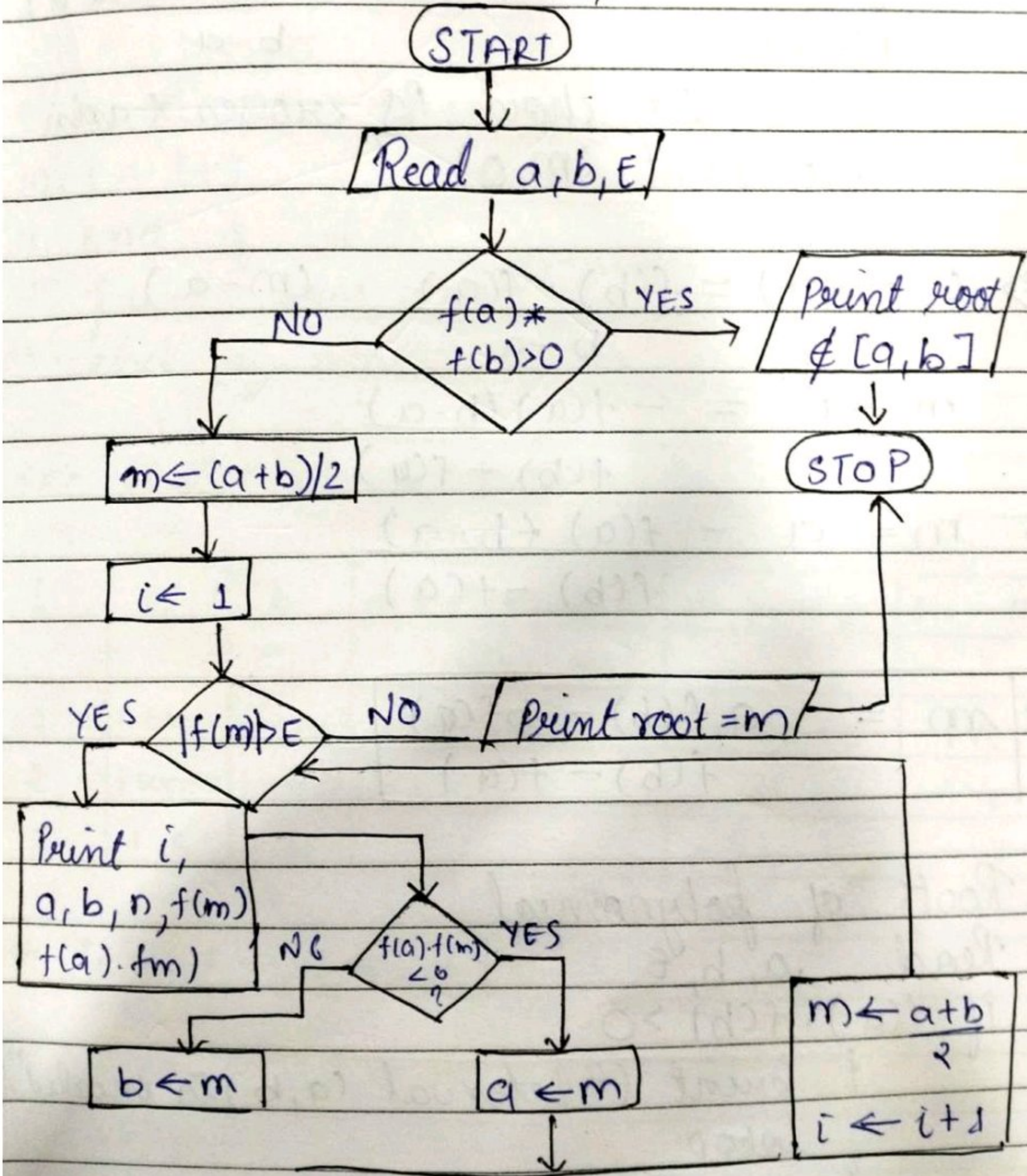
Bisection Method

⇒ Polynomial equation stored in computer -?

$Y = x^2 - x - 1$

0	-1
1	-1
2	1

Y



}
getch();
}

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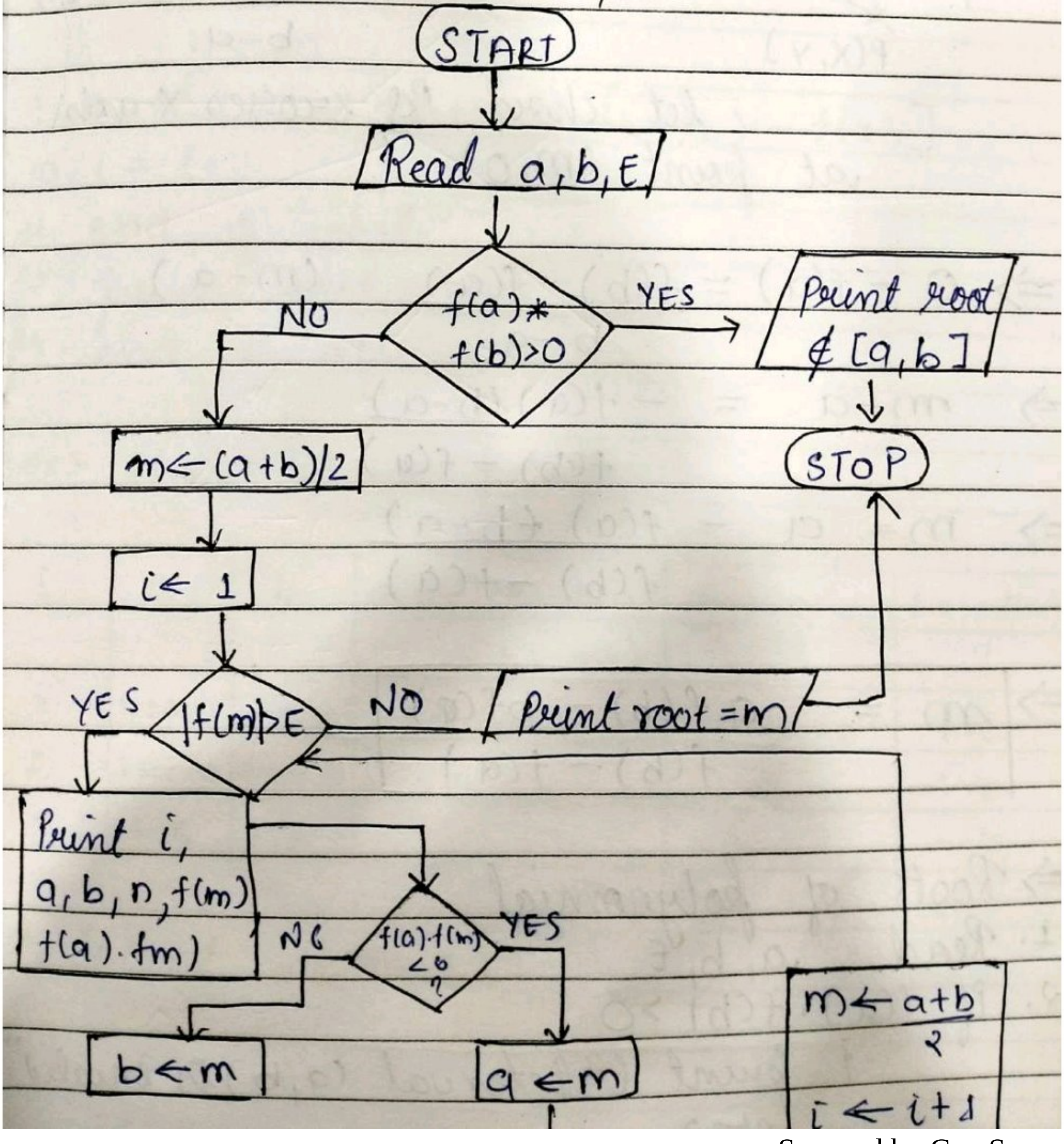
Bisection Method

⇒ Polynomial equation stored in computer :- ?

$Y = x^2 - x - 1$

0	−1
1	−1
2	1

Y



}
getch();
}

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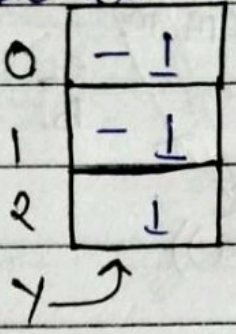
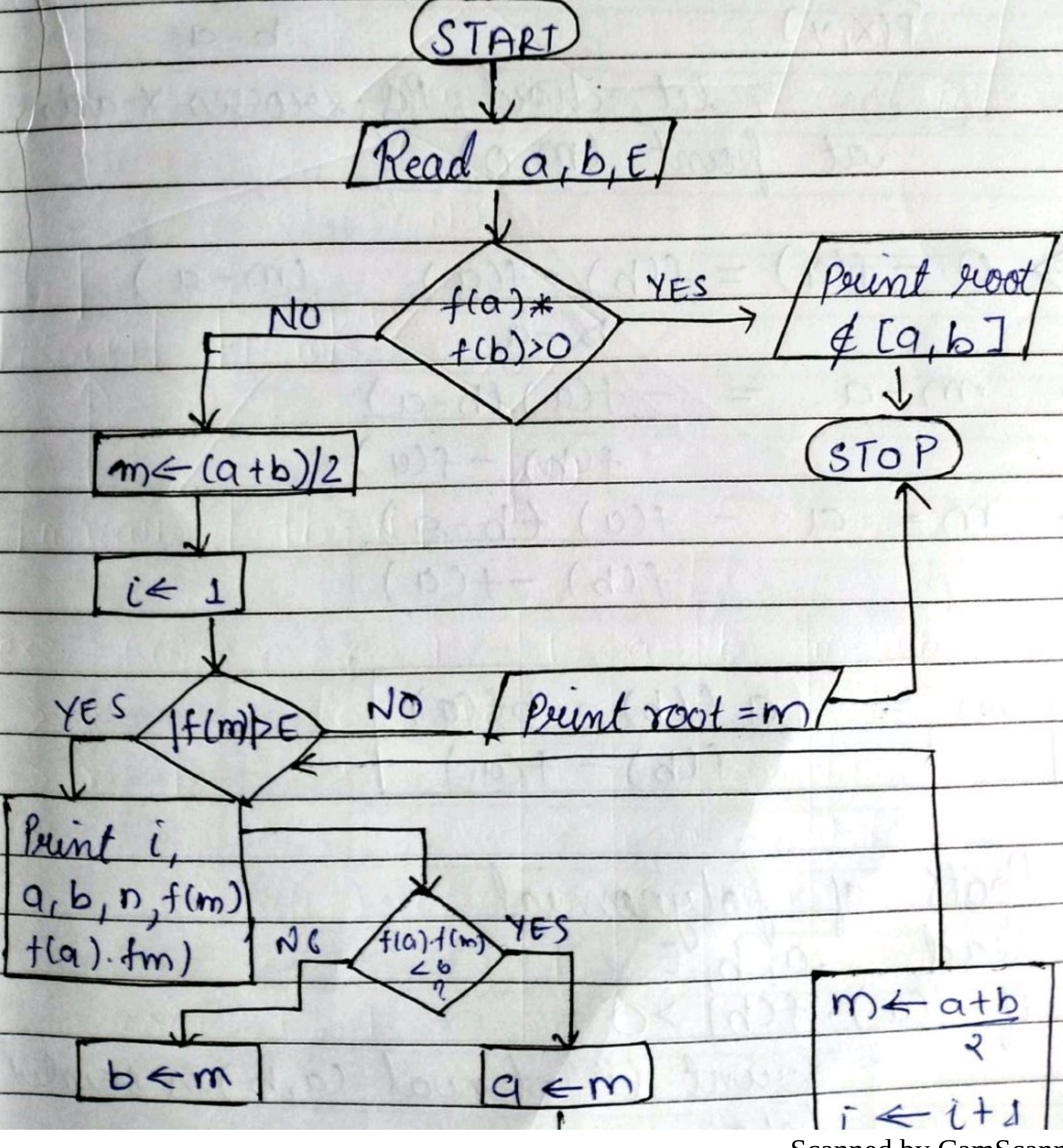
Bisection Method

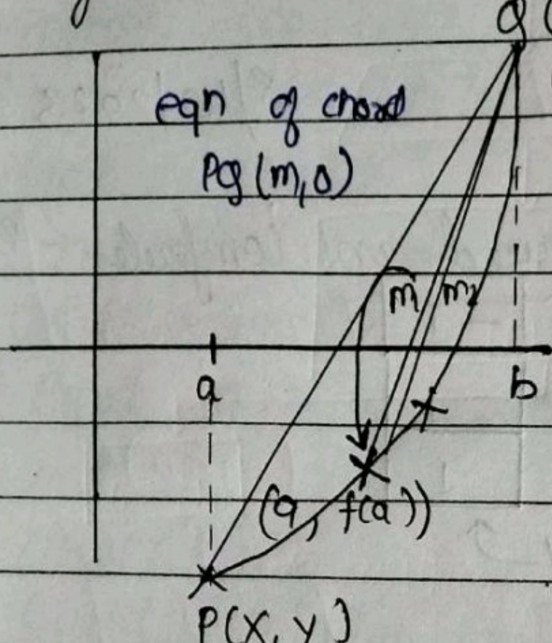
⇒ Polynomial equation stored in computer :- ?

$Y = x^2 - x - 1$





Regula falsi (Method of false position)



The eqn of line passing through P & Q

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - f(a) = \frac{f(b) - f(a)}{b - a}(x - a)$$

let chord PQ crosses x-axis at point $(m, 0)$

$$\Rightarrow 0 - f(a) = \frac{f(b) - f(a)}{b - a}(m - a)$$

$$\Rightarrow m - a = \frac{-f(a)(b - a)}{f(b) - f(a)}$$

$$\Rightarrow m = a - \frac{f(a)(b - a)}{f(b) - f(a)}$$

$$\Rightarrow \boxed{m = \frac{a f(b) - b f(a)}{f(b) - f(a)}}$$

$\Rightarrow$ Roots of polynomial

1. Read a, b, E
2. If $f(a) \cdot f(b) > 0$

   { print ("Interval (a, b) is invalid")

   stop

   }

3. otherwise

4. $m \leftarrow \dfrac{af(b) - bf(a)}{f(b) - f(a)}$

5. $i \leftarrow 1$
6. while $(|f(m)| > \epsilon)$
7. print $i, a, b, m, f(m), f(a) - f(m)$
8. if $(f(a) * f(m) > 0)$

   $a \leftarrow m$

   else

   $b \leftarrow m$
9. $m = [a f(b) - b f(a)] / [f(b) - f(a)]$
10. $i = i + 1$
11. end of while
12. print root $= m$
13. end of algo

ex:- $f(x) = x^2 - x - 1$

i	a	b	$m = \dfrac{af(b) - bf(a)}{f(b) - f(a)}$	$f(m) = m^2 - m - 1$	$f(a) - f(m)$
1	1.0000	2.000	1.50000	−0.2500	+ve
2	1.50000	2.0000	1.6		+ve
3	1.5	1.6			−ve

Secant method :-

(START)

↓

/Read a, b, E/

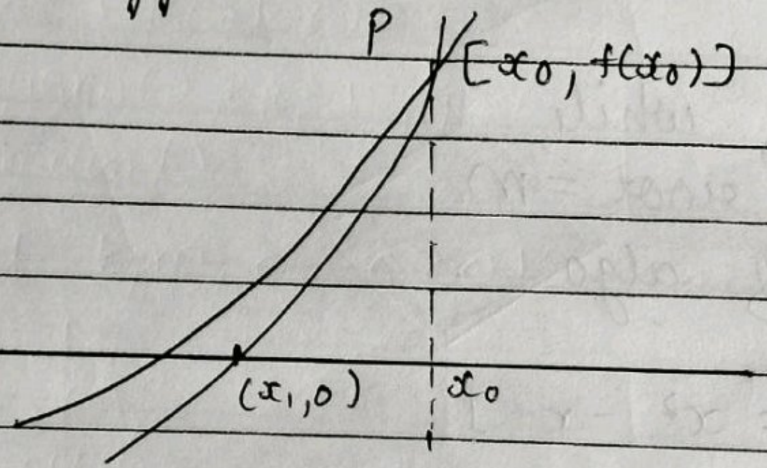
↓

$$m = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$i \leftarrow 1$ | Print [a, b]

★ Further Same as bisection method

Newton Raphson method :-



Equation of line passing through one point

$$\boxed{y - y_1 = f(x)(x - x_1)}$$

$$y - y_0 = f'(x_0)(x - x_0)$$

Let tangent cuts x-axis at point (x, 0)

$$\Rightarrow \quad 0 - y_0 = f'(x_0)(x_1 - x_0)$$

$$\Rightarrow \quad x_1 - x_0 = \frac{-f'(x_0)}{f'(x_0)}$$

$$\Rightarrow \quad \boxed{x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}}$$

$\Rightarrow \quad x_0 \leftarrow x_1$

$$\Rightarrow \quad x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$\Rightarrow \quad x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$\Rightarrow \quad x_{i+1} = x_i - \frac{f(x_1)}{f'(x_1)} \qquad m = b - \frac{f(b)}{f'(b)}$$

$f(x) = x^2 - x - 1$

i	b	$m = b - \frac{f(b)}{f'(b)}$	$f(m) = m^2 - m - 1$
1	2.000	$m = 1.66667$	$>E$
2	1.66667	$m = 1.6170$	$>E$
3	1.6170	$m = 1.6180$	$>E$
4	1.6180	$m = 1.6180$	$>E$

$\Rightarrow$ **Algorithm for Newton Raphson Method**

1. Read b, E
2. $m = b - \frac{f(b)}{f'(b)}$
3. $i = 1$
4. while $(|f(m)| > E)$
5. print $i, b, m, f(m)$
6. $b \leftarrow m$
7. $m \leftarrow b - f'(b)/f(b)$
8. $i \leftarrow i+1$

9. end of while
10. print root = m
11. end of algo

⇒ Program for Newton

```
#include <stdio.h>
#include <conio.h>
#include <math.h>
float f (float x)
{ return (x*x -x-1); }
float df (float x)
  { return (2*x-1); }
void main ()
 { float b,m, e = 0.001
   int i;
   clrscr();
   printf("\n Enter b :\n");
   scanf("%f", &b);
   m = b-f(b)/df(b);
   i=1
   while (fabs(f(m))>e)
   { printf ("\n i=%d \t b=%f\t
               m=%f \t f(m)=%f\n",
            i,b,m,f(m))
      b=m;
      m=b-f(b)/df(b);
```

```
printf ("root = %f ", m);
getch ();
}
}
```

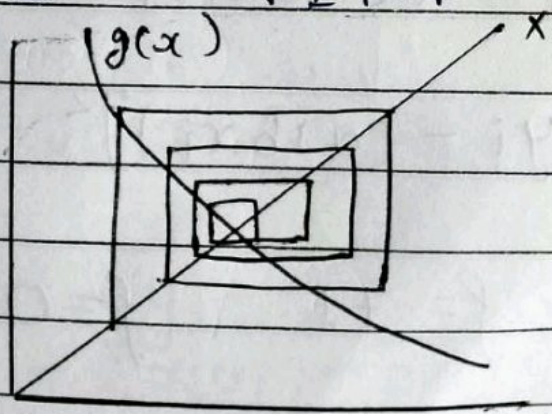
Fixed point Iterative method

$$x^2 - x - 1$$

$x = (1+x)^{0.5}$ $\quad$ $x = x^2 - 1$ $\quad$ $x = \frac{1+x}{x}$ $\quad$ $x = \frac{1}{x-1}$

$g(x) = (1+x)^{0.5}$	$g'(x) = 1/2\sqrt{1+x}$	$1/2\sqrt{2}$	$1/2\sqrt{3}$	$\lvert g'(x)\rvert < 1$
$g(x) = x^2 - 1$	$g'(x) = 2x$	> 1	> 1	
$g(x) = \frac{1+x}{x}$	$g'(x) = \frac{-1}{x^2}$	-1	$\frac{-1}{4}$	
$g(x) = \frac{1}{x-1}$	$g'(x) = \frac{1}{(x-1)^2}$	$-\infty$	-1	

1	b	$g(b) = \sqrt{1+x}$
1	1	$\sqrt{1+1}$
2	$\sqrt{2}$	$\sqrt{1+\sqrt{2}}$
3	$\sqrt{1+\sqrt{2}}$	$\sqrt{1+\sqrt{3}}$
4	$\sqrt{1+\sqrt{3}}$	$\sqrt{1+\sqrt{4}}$



$\lvert g[x] \rvert < 1$

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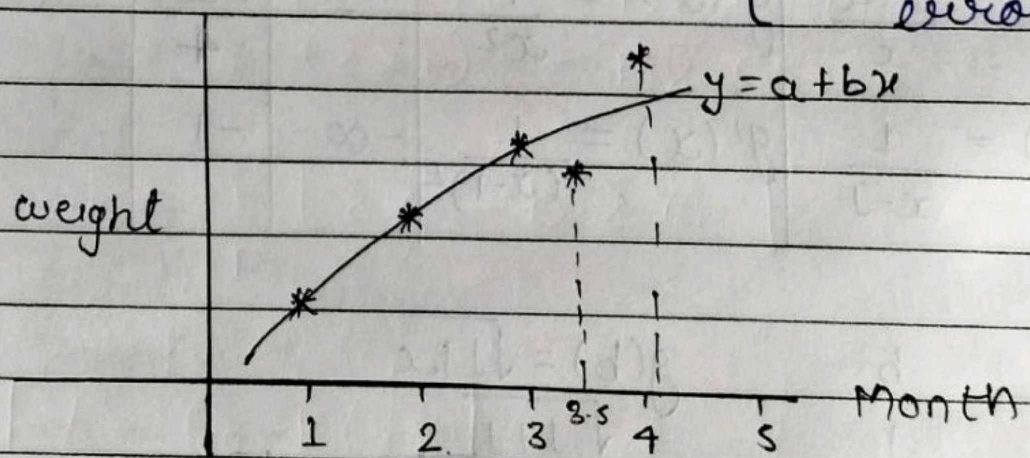
Data Prediction / forecasting

relations

$$\rho = \frac{cov(x,y)}{\sigma_x \sigma_y} = \frac{\sqrt{\frac{1}{n}(X-\bar{Y})(Y-\bar{Y})}}{\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}\sqrt{\frac{\Sigma(Y-\bar{Y})^2}{n}}}$$

$$\underbrace{0 < \rho < 0.5}_{\text{less related}} < \underbrace{\rho < 1}_{\text{Highly related}} \rightarrow \text{perfectly related}$$

deviation $U = \Sigma e_i^2 \rightarrow min^m$ { e_i = deviation of error



↳ data collection ↳ plotting data

$\Rightarrow \quad e_i = [y_1 - (a+bx_1)]^2$

$\Rightarrow \quad e_i = [y_i - (a+bx_i)]^2$

⇒ Scatter plot

$\rho = -1 \qquad \rho = +1 \qquad \rho = 0$

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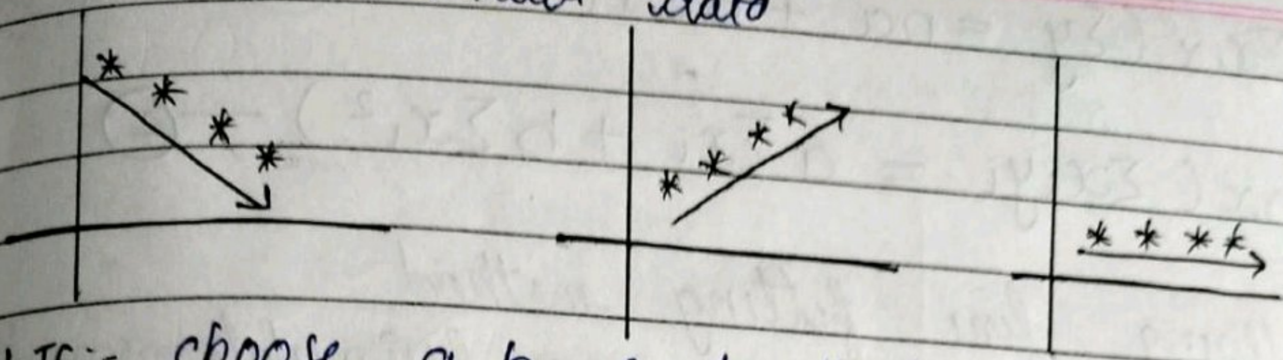
Linear data



NOTE:- choose a, b such that
$U = \sum [y_i - (a+bx_i)]^2$ should be minimum

→ Least square principle
at max & min → derivatives is zero

$\Rightarrow y = a + bx$

$\Rightarrow \frac{\partial U}{\partial a} = 0, \quad \frac{\partial U}{\partial b} = 0$

$\Rightarrow \frac{\partial U}{\partial a} = 2[y-(a+bx_i)]' \frac{\partial}{\partial a}\{[y-(a+bx_i)]\}$

$\Rightarrow \frac{\partial U}{\partial a} = 2[y-(a+bx_i)](-1)$

$\Rightarrow \frac{\partial U}{\partial a} = -2[y-a-bx_i]$

$\Rightarrow \frac{\partial U}{\partial a} = 2[y-(a+bx_i)]' \frac{\partial}{\partial b}\{[y-(a+bx_i)]\}$

$= 2[y-(a+bx_i)] \times -x_i$

$\Rightarrow \frac{\partial U}{\partial a} = -2\{x_i[y-a-bx_i]\}$

$\Rightarrow 0 = \Sigma y - \Sigma a - b\Sigma x$

$\Rightarrow 0 = \Sigma x_i y_i - a\Sigma x_i - b\Sigma x_i^2$

$$\Sigma x_i \times (\Sigma y = na + b\Sigma x_i) \text{ — } (1)$$

$$n \times (\Sigma x_i y_i = a\Sigma x_i + b\Sigma x_i^2) \text{ — } (2)$$

Using line fitting method
least square sum principle

$$\Sigma x_i \Sigma y_i = na\Sigma x_i + b(\Sigma x_i)^2$$

$$n\Sigma x_i y_i = an\Sigma x_i + bn\Sigma x_i^2$$

$$\Sigma x_i \Sigma y_i - n\Sigma x_i y_i = b[(\Sigma x_i)^2 - n\Sigma x_i^2]$$

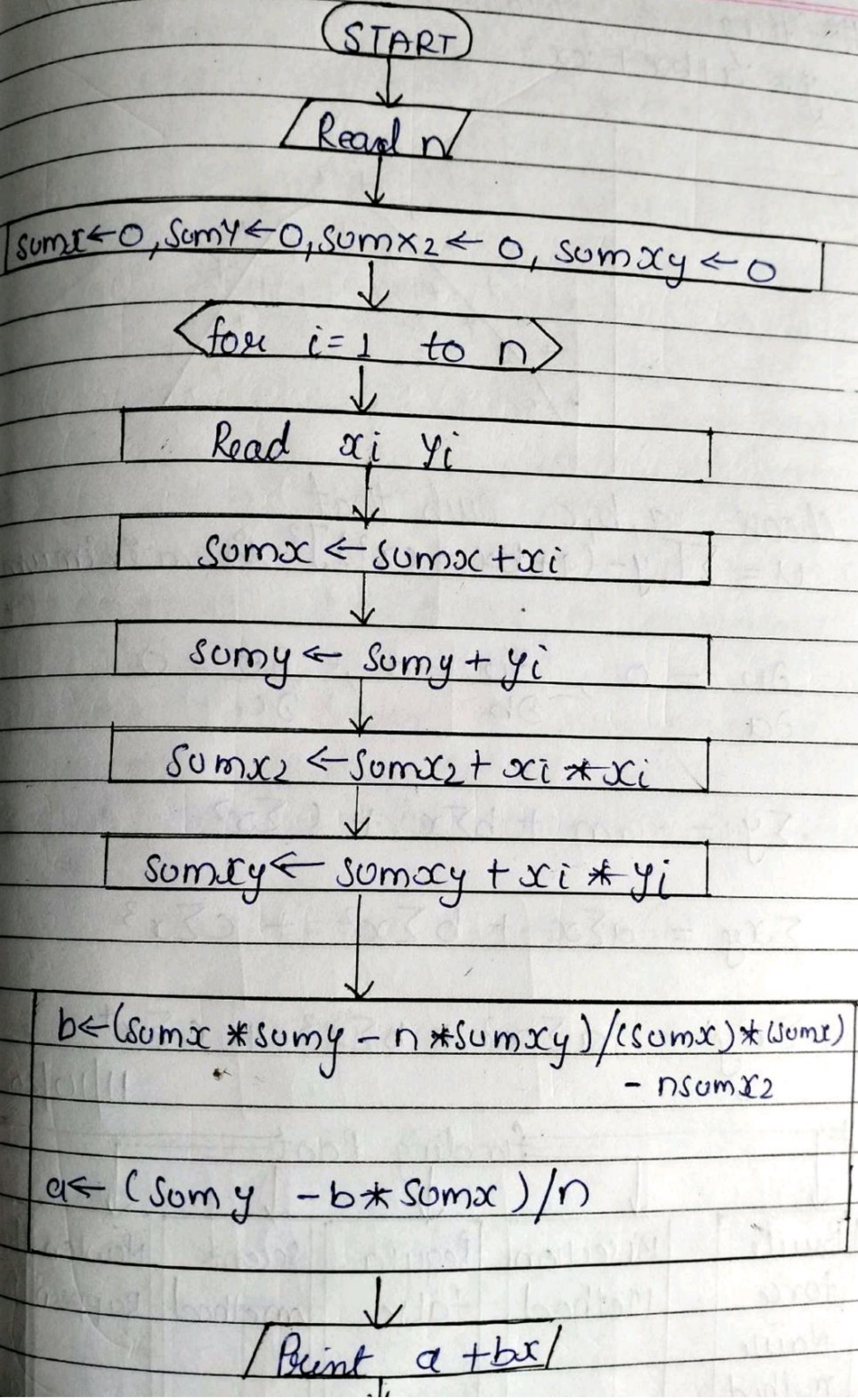
$$\Rightarrow b = \frac{\Sigma x_i \Sigma y_i - n\Sigma x_i y_i}{(\Sigma x_i)^2 - n\Sigma x_i^2}$$

$$\Rightarrow a = \frac{1}{n}[\Sigma y - b\,\Sigma x_i]$$

dependent

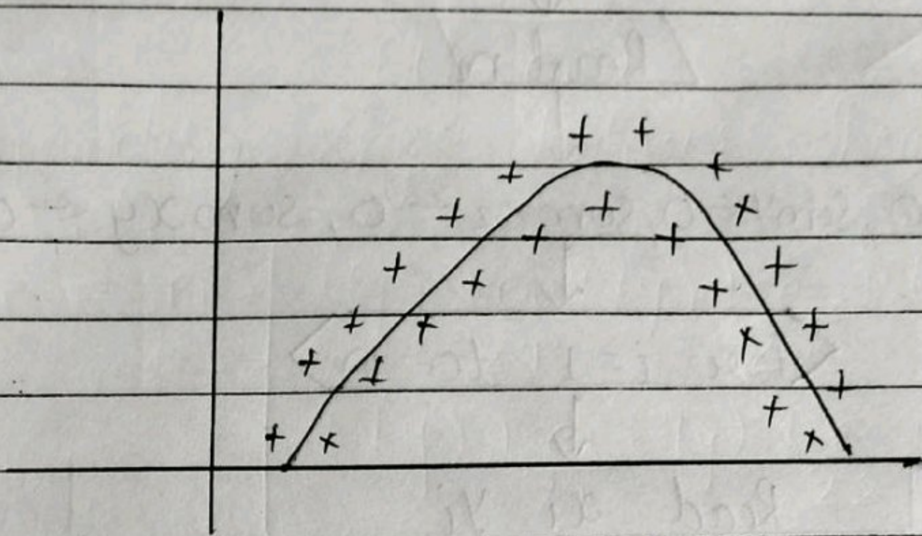
$$\boxed{Y = \underbrace{\theta_0 + \theta_1}_{\text{(coefficient)}} x}$$

⇒ Now let us draw a flow chart of solving this


START
Read n
sumx ← 0, sumy ← 0, sumx2 ← 0, sumxy ← 0
for i = 1 to n
Read xi yi
sumx ← sumx + xi
sumy ← sumy + yi
sumx2 ← sumx2 + xi * xi
sumxy ← sumxy + xi * yi
b ← (sumx * sumy − n * sumxy) / (sumx) * (sumx) − nsumx2
a ← (sumy − b * sumx) / n
Print a + bx

fitting of parabola (Non linear data)

$y = a + bx + cx^2$



choose a, b, c such that

$$U = \sum [y - (a + bx + cx^2)]^2 \text{ is minimum}$$

$$\frac{\partial U}{\partial a} = 0, \quad \frac{\partial U}{\partial b} = 0, \quad \frac{\partial U}{\partial c} = 0$$

$$\sum y = an + b\sum x + c\sum x^2$$

$$\sum xy = a\sum x + b\sum x^2 + c\sum x^3$$

$$\sum x^2 y = a\sum x^2 + b\sum x^3 + c\sum x^4$$

11/10/2023

finding Root

Brute force Naive method	Bisection Method	Regula false	Secent method	Newton Rappson	fixed Point interaction

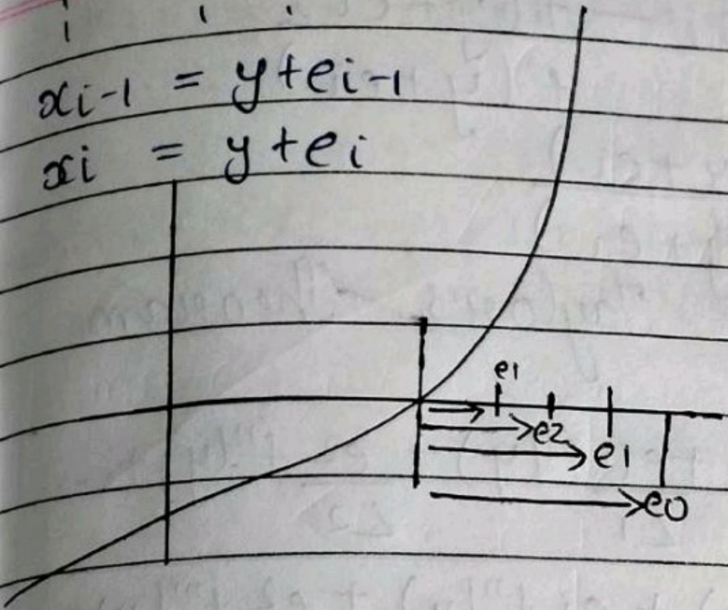
$x_0 = y+e_0$

$x_1 = y+e_1$

$\vdots$

$x_{i-1} = y+e_{i-1}$

$x_i = y+e_i$



$e_0 > e_1$

$e_1 > e_2$

$e_2 > e_3$ general form

$\Rightarrow \boxed{e_{i-1} > e_i}$

$\frac{e_1}{e_0} < 1$

$\frac{e_2}{e_1} < 1$

$\Rightarrow \boxed{\frac{e_i}{e_{i-1}} < 1}$ general form

$$\Rightarrow \lim_{i\to\infty} \frac{e_{i+1}}{e_i^{(k)}} < M$$

M = constant, k = order of convergence

Ques1) Prove that order of convergence of Newton Raphson Method is 2

Soln Let e_i is the error in computation of x_i

$$y+e_i = x_i \quad \text{——— (1)}$$

Let e_{i+1} is the error in computation of x_{i+1}

$$y+e_{i+1} = x_{i+1} \quad \text{——— (2)}$$

If y is actual root of $f(x)=0$ then $f(y) = 0$ ——— (3)

$x_{i+1} = x_i - f(x_i)$

$$\Rightarrow y + e_{i+1} = y + e_i - \frac{f(y+e_i)}{f'(y+e_i)}$$

$$\Rightarrow e_{i+1} = e_i - \frac{f(y+e_i)}{f'(y+e_i)}$$

$\because$ Expanding by Taylor's Theoram

$$\Rightarrow e_{i+1} = e_i - \left[\frac{f(y) + \frac{e_i}{\angle 1} f'(y) + \frac{e^2}{\angle 2} f''(y) + \cdots}{f''(y) + \frac{e_i}{\angle 1} f''(y) + \frac{e^2}{\angle 2} f'''(y) + \cdots}\right]$$

$\because$ Taking common $e_i f'(y)$

$$\Rightarrow e_{i+1} = e_i - \frac{e_i f'(y)\left[1 + \frac{e_i}{\angle 2}\frac{f''(y)}{f'(y)} + \frac{e_i^2}{\angle 3}\frac{f'''(y)}{f'(y)}\right]}{f'(y)\left[1 + \frac{e_i}{\angle 1}\frac{f''(y)}{f'(y)} + \frac{e_i^2}{\angle 2}\frac{f'''(y)}{f'(y)}\right]}$$

$\because$ Ignoring higher power of e_i

$$\Rightarrow e_{i+1} = e_i - \frac{e_i\left[1 + \frac{e_i f''(y)}{2f'(y)}\right]}{\left[1 + \frac{e_i f''(y)}{f'(y)}\right]}$$

$$\Rightarrow e_{i+1} = e_i - e_i\left[1 + \frac{e_i f''(y)}{2f'(y)}\right]\left[1 + \frac{e_i f''(y)}{f'(y)}\right]^{-1}$$

$\because$

$\because$ Expanding by binomal and ignoring higher power

$$\Rightarrow e_{i+1} = e_i - e_i\left[1 + \frac{e_i f''(y)}{2f'(y)}\right]\left[1 - \frac{e_i f''(y)}{f'(y)}\right]$$

$$\Rightarrow e_{i+1} = e_i - e_i\left[1 + \frac{e_i f''(y)}{2f'(y)} - \frac{e_i f''(y)}{f'(y)} - \frac{e_i^2 f''(y)}{2f'(x)}\right]$$

α

Ignoring this term

$$\Rightarrow e_{i+1} = e_i - e_i\left[1 + \frac{\alpha}{2} - \alpha\right]$$

$$= e_i - e_i\left[1 - \frac{\alpha}{2}\right]$$

$$= e_i - e_i\left[1 - \frac{e_i f''(y)}{2f'(y)}\right]$$

$$= \cancel{e_i - e_i} + \frac{e_i^2 f''(y)}{2f'(y)}$$

$$\Rightarrow e_{i+1} = \frac{f''(y)}{2f'(y)}$$

Comparing it with

$$\Rightarrow \boxed{\lim_{i\to\infty} \frac{e_{i+1}}{e_i^2} \le M} \quad \lim_{i\to\infty} \frac{e_{i+1}}{e_i^K} \le M$$

$$\Rightarrow \boxed{K=2}$$

it shows that $e_{i+1} \propto e^K$

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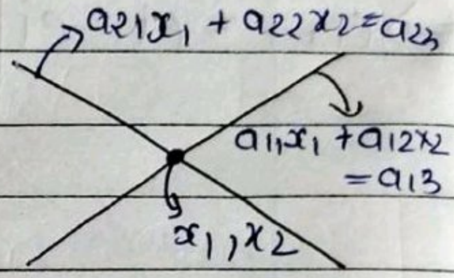
System of Simultaneous Eqn

→ One dimension :- $ax_1 = a'$

→ Two dimension :-

$a_{11}x_1 + a_{12}x_2 = a_{13}$

$a_{21}x_1 + a_{22}x_2 = a_{23}$



This will give point of intersection

→ Three dimension :-

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = a_{14}$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = a_{24}$

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = a_{34}$

firstly it will give line and then after we will get point of intersection

★ We will represent them in the form of matrix

① for 2D :- $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a_{13} \\ a_{23} \end{bmatrix}$

⇒ Augmented matrix

$$\left[\begin{array}{cc|c} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{array}\right]$$

② for 3D :- $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \begin{bmatrix} a_{14} \\ a_{24} \\ a_{34} \end{bmatrix}$

⇒ Augmented matrix

$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{array}\right]$$

So now our general system of eq^n will be

$$a_{11}x_1 + a_{12}x_1 + a_{13}x_3 + \cdots\cdots a_{1n}x_n = a_{1(n+1)}$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots\cdots a_{2n}x_n = a_{2(n+1)}$$
$$\vdots \quad \vdots \quad \vdots \qquad\qquad \vdots \quad \vdots$$
$$a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \cdots\cdots a_{nn}x_n = a_{n(n+1)}$$

Now the matrix will look like this

$$\left[\begin{array}{ccccc|c} a_{11} & a_{12} & a_{13} & \cdots\cdots & a_{1n} & a_{1(n+1)} \\ a_{21} & a_{22} & a_{23} & \cdots\cdots & a_{2n} & a_{2(n+1)} \\ a_{31} & a_{32} & a_{33} & \cdots\cdots & a_{3n} & a_{3(n+1)} \\ \vdots & \vdots & \vdots & & \vdots & \\ a_{n1} & a_{n2} & a_{n3} & \cdots\cdots & a_{nn} & a_{n(n+1)} \end{array}\right]$$

Now using a diagonal elimination (Guass elimination method by which we'll make all the element below diagonal 0

→ Column ①

$R_2 \leftarrow R_2 - R_1\left(\frac{a_{21}}{a_{11}}\right)$ | $R_2 \leftarrow R_2 - R_1 * U,\ U = \frac{a_{21}}{a_{11}}$

$R_3 \leftarrow R_3 - R_1\left(\frac{a_{31}}{\ }\right)$ | $R_3 \leftarrow R_2 - R_1 * U,\ U = \frac{a_{31}}{a_{11}}$

$R_4 \leftarrow R_4 - R_1 \left(\frac{a_{41}}{a_{11}}\right)$ | $R_4 \leftarrow R_4 - R_1 * U$, $U = \frac{a_{41}}{a_{11}}$

$R_n \leftarrow R_n - R_1 \left(\frac{a_{n1}}{a_{11}}\right)$ | $R_n \leftarrow R_n - R_1 * U$, $U = \frac{a_{n1}}{a_{11}}$

after doing this we got

→Pivot

$$\left[\begin{array}{cccccc|c} a_{11} & a_{12} & a_{13} & - & - & a_{1n} & a_1(n+1) \\ 0 & a_{22}' & a_{23} & - & - & a_{2n} & a_2(n+1)' \\ 0 & a_{32}' & a_{33} & - & - & a_{3n} & \\ \vdots & \vdots & \vdots & & & & \\ 0 & a_{n2}' & a_{n3}' & - & - & a_{nn}' & a_n(n+1)' \end{array}\right]$$

→ Column ② :- now our pivot element is a_{22}'

$R_3 \leftarrow R_3 - R_2 \left(\frac{a_{32}'}{a_{22}'}\right)$

$R_4 \leftarrow R_4 - R_2 \left(\frac{a_{42}'}{a_{22}'}\right)$

$\vdots \quad \vdots \quad \vdots$

$R_n \leftarrow R_n - R_2 \left(\frac{a_{n2}'}{a_{22}}\right)$

perform this until we reach last column

⇒ Algo for doing this

① Read n

② for i = 1 to n by 1

for j = 1 to (n+1) by 1

Read a_{ij}

end of j

end of i

$$a_{ij} = 0 \begin{cases} a_{ij} & i \le j \\ 0 & i > j \end{cases}$$

③ for $k = 1$ to $(n-1)$ by 1

for $i = (k+1)$ to n by 1

$u \leftarrow \frac{a_{ik}}{a_{kk}}$

for $j = k$ to $(n+1)$ by 1

$a_{ij} \leftarrow a_{ij} - u * a_{kj}$

end of j

end of i

end of k

④ $x_n \leftarrow a_{n(n+1)} / a_{nn}$

⑤ for $i = (n-1)$ to 1 by -1

sum $\leftarrow 0$

for $j = i+1$ to n by 1

sum $\leftarrow$ sum $+ a_{ij} \cdot x_j$

end of j

end of i

⑥ for $i = 1$ to n

⇒ In short this algo will look like,

Read
Upper Triangular Matrix
Back Substitution

★ There is another way of doing this - and that is to make both the upper and lower element of diagonal 0 and this method is known as Gauss Jordan method

14/10/2023

Ques 1)

$$2x_1 - 2x_2 = 1$$
$$-x_1 + 2x_2 - 3x_3 = -2$$
$$-2x_2 + 2x_3 - 4x_4 = -1$$
$$x_3 - x_4 = 3$$

Soln

$$2x_1 - 2x_2 + 0x_3 + 0x_4 = 1$$
$$-x_1 + 2x_2 - 3x_3 + 0x_4 = -2$$
$$0x_1 + -2x_2 + 2x_3 - 4x_4 = -1$$
$$0x_1 + 0x_2 + x_3 - x_4 = 3$$

⇒ Augmented matrix will be

$$A:B = \left[\begin{array}{cccc|c} 2 & -2 & 0 & 0 & 1 \\ -1 & 2 & -3 & 0 & -2 \\ 0 & -2 & 2 & -4 & -1 \\ 0 & 0 & 1 & -1 & 3 \end{array}\right]$$

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$R_2 \leftarrow R_2 - R_1 \left(\frac{a_{21}}{a_{11}}\right) \Rightarrow R_2 - R_1 \left(\frac{-1}{2}\right)$

$\Rightarrow R_2 \leftarrow R_2 - \frac{R_1}{2}$

$$\left[\begin{array}{cccc|c} 2 & -2 & 0 & 0 & 1 \\ 0 & 1 & -3 & 0 & -3/2 \\ 0 & -2 & 2 & -4 & -1 \\ 0 & 0 & 1 & -1 & 3 \end{array}\right]$$

$\Rightarrow R_3 \leftarrow R_3 - R_2 \left(\frac{-2}{1}\right) \Rightarrow R_3 \leftarrow R_3 + R_2$

$$\left[\begin{array}{cccc|c} 2 & -2 & 0 & 0 & 1 \\ 0 & 1 & -3 & 0 & -3/2 \\ 0 & 0 & -4 & -4 & -4 \\ 0 & 0 & 1 & -1 & 3 \end{array}\right]$$

$\Rightarrow R_4 \leftarrow R_4 - R_3 \left(\frac{1}{-4}\right) \Rightarrow R_4 \leftarrow R_4 + \frac{R_3}{4}$

$$\left[\begin{array}{cccc|c} 2 & -2 & 0 & 0 & 1 \\ 0 & 1 & -3 & 0 & -3/2 \\ 0 & 0 & -4 & -4 & -4 \\ 0 & 0 & 0 & -2 & 2 \end{array}\right]$$

This is upper triangular matrix
Hence modified system of eq^n will be

$2x_1 - 2x_2 = 1$ ——— (1)

$x_2 - 3x_3 = -3/2$ ——— (2)

$$x_3 + x_4 = 1 \quad \text{——— (3)}$$
$$-x_4 = 1 \quad \text{——— (4)}$$

from eqn $\boxed{x_4 = -1}$

put $x_4 = -1$ in eq^n (3)

$\Rightarrow x_3 - 1 = 1$

$\Rightarrow \boxed{x_3 = 2}$

put x_3 in eq^n (2)

$$x_2 = 3x_3 - \frac{3}{2}$$

$$x_2 = 6 - \frac{3}{2}$$

$$\boxed{x_2 = 9/2}$$

put x_2 in eq^n (1)

$$2x_1 - 2x_2 = 1$$

$$2x_1 - \left(\cancel{2}\,\frac{9}{\cancel{2}}\right) = 1$$

$$2x_1 = 1 + 9$$

$$\boxed{x_1 = 5}$$

We can also solve it by Jacobi method

	initial	
x_1	0	1/2
x_2	0	-1
x_3	0	-1/2
x_4	0	[illegible]

In this method firstly we put 0 in all the eq^n after putting that we got new values from x_1 to x_4 then we'll put those new value we'll continue to do this until any two coloumn becomes same or almost same

There is another method named Gauss saddle in which when we got value of x_1 then we will put new value of x_1 rather than putting 0. and we'll continue to do this until any two column becomes same or almost same.

15/10/2023

System of simultaneous equation

$$Ax = B$$

$$\downarrow$$

$$[A : B]$$

- Direct methods
  - Gauss elimination
    - → upper triangular
    - → Back substitution
  - Gauss jordan
    - → diagonalization
    - → direct root
- Iterative method
  - Jacobi
    - → uses previous values to compute
  - Gauss seidel
    - → use latest values to

$\Rightarrow a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = a_{14}$

$\Rightarrow a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = a_{24}$

$\Rightarrow a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = a_{34}$

$x_1 = \frac{1}{a_{11}}[a_{14} - \text{Sum}]$

$\text{Sum} = a_{12}x_2 + a_{13}x_3$

→ pivoting (Row interchange only)
- partial
- complete
  ↳ Row 2 Column Interchange

→ Pivot condensation algo
→ switch/adjust Rows to make a_{11} highest

$|a_{mk}|$ for $m = k+1$ to n

→ Ill Conditioned equation :-

$ax + by = c$

$2.500x + 5.200y = 6.200$ $\quad$ $2.500x + 5.200y = 6.20$

$1.251x + 2.605y = 3.152$ $\quad$ $1.251y + 2.606y = 3.15$

$X = -0.32794E2$ $\quad$ $X = -0.3217E2$ $\quad$ $X = 0.2352E2$

$Y = -0.16958E2$ $\quad$ $Y = 0.1666E2$ $\quad$ $Y = 0.1250E2$

1% change $\quad$ 30% change

⇒ Algorithm for iterative methods

(1) for $i = 1$ to n by 1
  for $j = 1$ to $n+1$
    Read a_{ij}

(2) Read e, maxit

(3) for $i = 1$ to n by 1
    $x_i \leftarrow 0$
  end for

(4) for iter = 1 to maxit do
    big ← 0

for i=1 to n by 1
    sum ← 0
    for j=1 to n by 1 do
        if (j ≠ i)
            sum ← sum + $a_{ij} x_j$
    temp ← (a_{in+1} − sum)/a_{ii}
    rel error ← |(x_i − temp)/temp|
    x_i ← temp
end for
if (big <= e) then
    begin / Write converge to a solⁿ
    for i=1 to n by 1
        print x_i
    print do not converge
else
    for i=1 to n by 1
        print x_i

Derivation for ILL Conditioned eqⁿ

↪ $ax + by = c$ ——— (1)

⇒ $px + qy = r$ ——— (2)

⇒ $x = (c - by)/a$

put in (2)

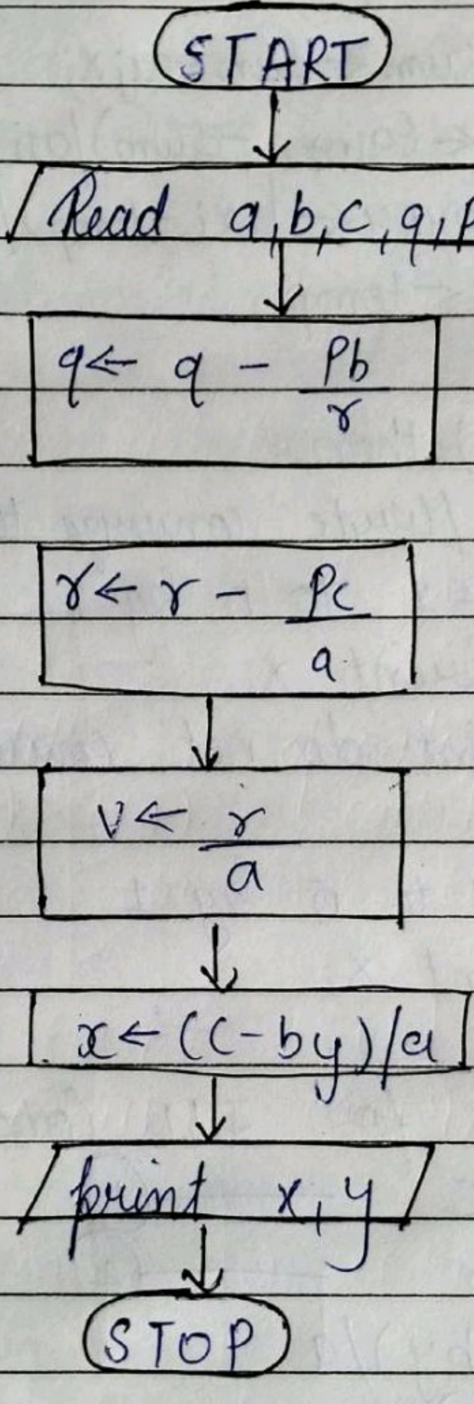
⇒ $p\,(c - by)/a + qy = r$

⇒ $\left(q - \frac{pb}{a}\right) y = r - \frac{pc}{a}$

$q \leftarrow q - \frac{pb}{a}, \quad r = r - \frac{pc}{a}$

$\Rightarrow ay = r$ or $y = \frac{r}{a}$

Flowchart for Ill conditioned eqn



A) $a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = a_{14}$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = a_{24}$

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = a_{34}$

$x_1^{(1)}$

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$x_2^{(1)}$
$x_3^{(1)}$

B) $a_{11}x_1^{(1)} + a_{12}x_2^{(1)} + a_{13}x_3^{(1)} = a_{14}^{(1)}$
$a_{21}x_1^{(1)} + a_{22}x_2^{(1)} + a_{23}x_3^{(1)} = a_{24}^{(1)}$
$a_{31}x_1^{(1)} + a_{32}x_2^{(1)} + a_{33}x_3^{(1)} = a_{34}^{(1)}$

(A) - (B)

$$\left[\begin{array}{l} a_{11}(x_1 - x_1^{(1)}) + a_{12}(x_2 - x_2^{(1)}) + a_{13}(x_3 - x_3^{(1)}) = \\ \qquad a_{14} - a_{14}^{(1)} \\ a_{21}(x_1 - x_1^{(1)}) + a_{22}(x_2 - x_2^{(1)}) + a_{23}(x_3 - x_3^{(1)}) = \\ \qquad a_{24} - a_{24}^{(1)} \\ a_{31}(x_1 - x_1^{(1)}) + a_{32}(x_2 - x_2^{(1)}) + a_{33}(x_3 - x_3^{(1)}) \\ \qquad = a_{34} - a_{34}^{(1)} \end{array}\right]$$

Now $e_1^{(1)} = x_1 - x_1^{(1)}$ | $a_{14} - a_{14}^{(1)} = \alpha_{14}$
$e_2^{(1)} = x_2 - x_2^{(1)}$ | $a_{24} - a_{24}^{(1)} = \alpha_{24}$
$e_3^{(1)} = x_3 - x_3^{(1)}$ | $a_{34} - a_{34}^{(1)} = \alpha_{34}$

$a_{11}e_1^{(1)} + a_{12}e_2^{(1)} + a_{13}e_3^{(1)} = \alpha_{14}^{(1)}$
$a_{21}e_1^{(1)} + a_{22}e_2^{(1)} + a_{23}e_3^{(1)} = \alpha_{24}^{(1)}$
$a_{31}e_1^{(1)} + a_{32}e_2^{(1)} + a_{33}e_3^{(1)} = \alpha_{34}^{(1)}$

$$\left.\begin{array}{l} x_1^{(2)} = e_1^{(1)} + x_1^{(1)} \\ x_2^{(2)} = e_2^{(1)} + x_2^{(1)} \\ x_3^{(2)} = e_3^{(1)} + x_3^{(1)} \end{array}\right\} \text{Refinement}$$

{Again put on equation and iterate until
LHS = RHS in eqn

⇒ Finite difference calculus :-

$y = f(x)$

dependent variable

Independent variable (x)

" equidistant " values like

Arguments : $a, a+h, a+2h, a+3h, \ldots$

$\ldots a+(n-1)h, a+nh$

Corresponding Y values — Entry :- $f(a), f(a+h), f(a+2h), \ldots$

$\ldots f(a+(n-1)h), f(a+nh)$

h is any fixed value

⇒ Forward difference operator "Δ"

$f(a+h) - f(a)$ is called forward difference of $f(a)$, denoted by $\Delta f(a)$

$$\boxed{\Delta f(a) = f(a+h) - f(a)}$$

Similarly, $\Delta f(a+h) = f(a+2h) - f(a+h)$

$\vdots$

$\Delta f(a+\overline{n-1}h) = f(a+nh) - f(a+\overline{n-1}h)$

→ Second forward difference :- "Δ^2"

Forward difference of $\Delta f(a) = \Delta^2 f(a)$

$\because \Delta^2 f(a) = \Delta [\Delta f(a)]$

$= \Delta [f(a+h) - f(a)]$

$= \Delta f(a+h) - \Delta f(a)$

$= f(a+2h) - f(a+h) - f(a+h) + f(a)$

$$\Rightarrow \Delta^2 f(a) = f(a+2h) - 2f(a+h) + f(a)$$

→ Third forward difference :- $\Delta^3 f(a)$

$$\Delta^3 f(a) = \Delta[\Delta^2 f(a)]$$
$$= \Delta[f(a+2h) - 2f(a+h) + f(a)]$$
$$= \Delta f(a+2h) - 2\Delta f(a+h) + \Delta f(a)$$
$$= f(a+3h) - f(a+2h) - 2[f(a+2h) - f(a+h)] + \Delta f(a)$$

$$\{\because \Delta f(a) = f(a+h) - f(a)\}$$

$$\Rightarrow \boxed{\Delta^3 f(a) = f(a+3h) - 3f(a+2h) + 3f(a+h) - f(a)}$$

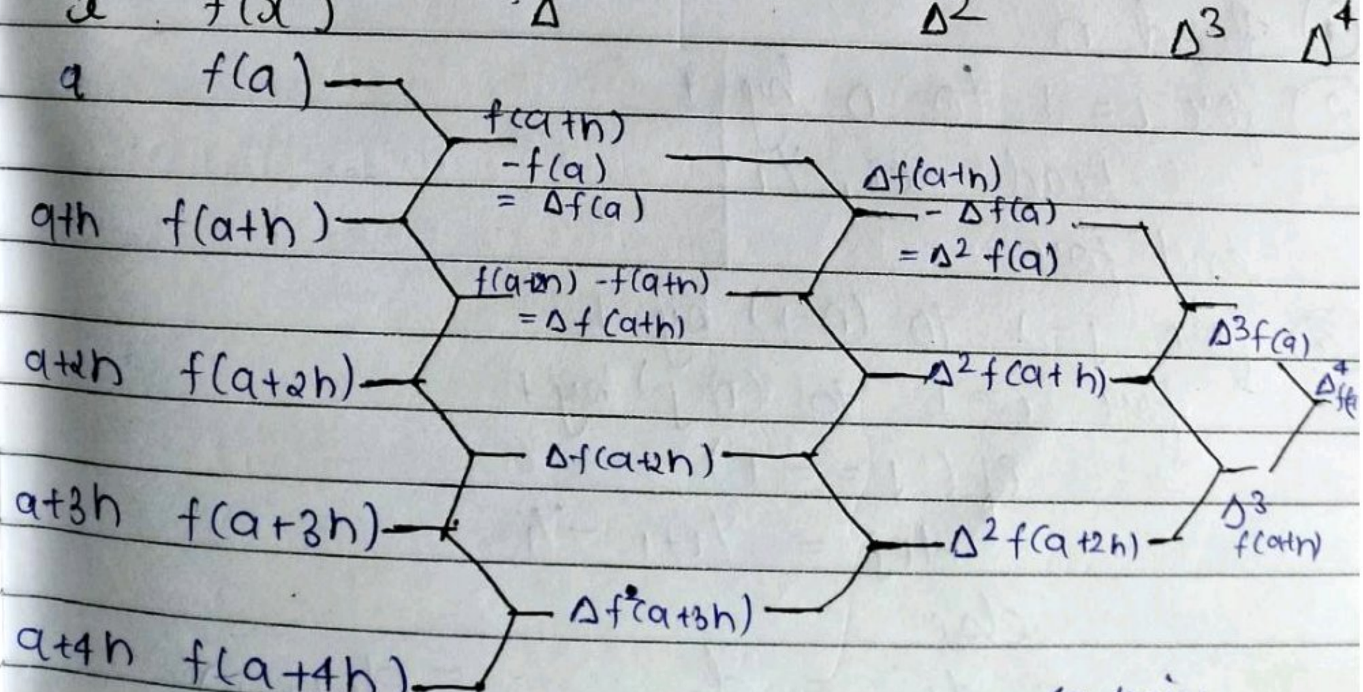
1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1



continue

$a+6h$ — $\Delta f(a+5h)$ — $\Delta^2 f(a+4h)$ — $\Delta^3 f(a+3h)$ — $\Delta^4 f(a+3h)$

$a+7h$ — $\Delta f(a+6h)$ — $\Delta^2 f(a+5h)$ — $\Delta^3 f(a+4h)$

Backward Difference Calculus (∇)

$\nabla \rightarrow$ nebla

Construction of difference table

d[5] [5]

	x	y					
1	$a=x_1$	$f(a)$	y_2-y_1	$d_{21}-d_{11}$	$d_{22}-d_{12}$	$d_{23}-d_{13}$	X
2	$a+h$	$f(a+h)$	y_3-y_2	$d_{31}-d_{21}$	$d_{32}-d_{2}$	X	X
3	$a+2h$	$f(a+2h)$	y_4-y_3	$d_{41}-d_{31}$	X	X	X
4	$a+3h$	$f(a+3h)$	y_5-y_4	X	X	X	X
5	$a+4h$	$f(a+4h)$	X	X	X	X	X

1) Read n

2) for $i = 1$ to n by 1

    Read x_i, y_i

   end for

3) for $j = 1$ to $(n-1)$ by 1

    for $i = 1$ to $(n-j)$ by 1

        if (j = = 1)

end if

end for

end for

Backward difference calculus

→ Backward difference operator "∆"

let $y = f(x)$ is a function of x, x takes equidistant values $f(a) - f(a-h)$ is called Backward difference of $f(a)$, denoted by $\Delta f(a) / \nabla f(a)$

$$\Rightarrow \nabla f(a) = f(a) - f(a-h)$$

$$\nabla^2 f(a) = \nabla[\nabla f(a)]$$

$$= \nabla[f(a) - f(a-h)]$$

$$= \nabla f(a) - \nabla f(a-h)$$

$$= f(a) - f(a-h) - [f(a-h) - f(a-2h)]$$

$$\Rightarrow \boxed{\nabla^2 f(a) = f(a) - 2f(a-h) + f(a-2h)}$$

$$\Rightarrow \boxed{\nabla^3 f(a) = f(a) - 3f(a-h) + 3f(a-2h) - f(a-3h)}$$

Construction of Backward difference table

X	Y		1	2	3	4	5
a+4h	f(a+4h)		×	×	×	×	×
a+3h	f(a+3h)		$Y_2 - Y_1$	×	×	×	×
a+2h	f(a+2h)		$Y_3 - Y_2$	$d_{31} - d_{21}$	×	×	×
a+h	f(a+h)		$Y_4 - Y_3$	$d_{41} - d_{31}$	$d_{42} - d_{32}$	×	×
a	f(a)		$Y_5 - Y_4$	$d_{51} - d_{41}$	$d_{52} - d_{42}$	$d_{53} - d_{43}$	×

$\nabla f(a)$ $\nabla^2 f(a)$ $\nabla^3 f(a)$ $\nabla^4 f(a)$

for $i = j+1$ to n by 1
    if $(j == 1)$
        $d_{ij} \leftarrow y_i - y_{i-1}$
    else
        $d_{ij} \leftarrow d_{i(j-1)} - d_{(i-1)(j-1)}$

Logic of Backward difference

→ Divided difference calculus (Δ)

⇒ $y = f(x)$

$x_0, x_1, x_2, x_3, \ldots\ldots x_{i-1}, x_i \ldots$

$(x_1 - x_0) \neq (x_2 - x_1) \neq (x_{i-1} - x_i)$

⇒ $\underset{x_0\, x_1}{\Delta} f(x) = f(x_0, x_1) = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$

⇒ $\underset{x_0\, x_1, x_2}{\Delta^2} = f(x_0, x_1, x_2) = \dfrac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0}$

$= \dfrac{f(x_2) - f(x_1)}{x_2 - x_1} - \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$

★ Shift operator

$E f(x) = f(x+h)$	$E^{-1} f(x) = f(x-h)$
$E^2 f(x) = f(x+2h)$	$E^{-2} f(x) = f(x-2h)$
$E^3 f(x) = f(x+3h)$	$E^{-3} f(x) = f(x-3h)$

$\Delta f(x) = f(x+h) - f(x)$

$\nabla f(x) = f(x) - f(x-h)$

x	y				
x_0	y_0				
		$\frac{y_1 - y_0}{x_1 - x_0} = y_0'$			
x_1	y_1		$\frac{y_1' - y_0'}{x_2 - x_0} = y_0''$		
		$\frac{y_2 - y_1}{x_2 - x_1} = y_1'$		$\frac{y_1'' - y_0''}{x_3 - x_0} = y_0'''$	
x_2	y_2		$\frac{y_2' - y_1'}{x_3 - x_1} = y_1''$		$\frac{y_1''' - y_0'''}{x_4 - x_0}$
		$\frac{y_3 - y_2}{x_3 - x_2} = y_2'$		$\frac{y_2'' - y_1''}{x_4 - x_1} = y_1'''$	
x_3	y_3		$\frac{y_3' - y_2'}{x_4 - x_2} = y_2''$		
		$\frac{y_4 - y_3}{x_4 - x_3} = y_3'$			
x_4	y_4				

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$\Rightarrow y = f(x)$

$a, a+h, a+2h, a+3h \ldots\ldots$

$f(a), f(a+h), f(a+2h), f(a+3h), \ldots\ldots$

$\boxed{\Delta f(x) = f(x+h) - f(x)}$ forward refference operator.

$\boxed{\nabla f(x) = f(x) - f(x+h)}$ Backward refference operator.

$\boxed{\delta f(x) = f(x+h/2) - f(x-h/2)}$ central diff. operator

$\boxed{\mu f(x) = \frac{f(x+h/2) + f(x-h/2)}{2}}$ Average operator

$\boxed{Ef(x) = f(x+h)}$ or $\boxed{E^{n} f(x) = f(x-h)}$ Shift operator

Divided difference operator [when arguments are not equidistant] :-

$$\boxed{\underset{x_0 x_1}{\Delta} f(x) = f(x_0, x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}}$$

⇒ Relationship among operators

$$\Delta f(x) = f(x+h) - f(x)$$
$$\Delta f(x) = E f(x) - f(x)$$
$$\Delta f(x) = (E-1) f(x)$$

$\Rightarrow \boxed{\Delta = E - 1}$ $\Rightarrow \boxed{E = 1 + \Delta}$

Similarly,

$$\nabla f(x) = f(x) - f(x-h)$$
$$\nabla f(x) = \Delta f(x-h)$$
$$\nabla f(x) = \Delta E^{-1} f(x)$$

$\Rightarrow \boxed{\nabla = \Delta E^{-1}}$

Similarly

$$\nabla f(x) = f(x) - f(x-h)$$
$$\nabla f(x) = f(x) - E^{-1} f(x)$$
$$\nabla f(x) = (1 - E^{-1}) f(x)$$

$\Rightarrow \boxed{\nabla = 1 - E^{-1}}$ $\Rightarrow \boxed{E^{-1} = 1 - \nabla}$ $\Rightarrow \boxed{E = (1-\nabla)^{-1}}$

we know that

$$\delta f(x) = f(x+h/2) - f(x-h/2) \quad \text{——— (1)}$$

and

$$\mu f(x) = \frac{f(x+h/2) - f(x-h/2)}{2} \quad \text{——— (2)}$$

$$2\mu f(x) = f(x+h/2) - f(x-h/2)$$

let $A = f(x+h/2)$ and $B = f(x-h/2)$

{we know that $(A+B)^2 - (A-B)^2 = 4AB$ }

putting values of A and B

$\Rightarrow [f(x+h/2) + f(x-h/2)]^2 - [f(x+h/2) - f(x-h/2)]^2$
$= 4\, f(x+h/2)\, f(x-h/2)$

putting values from eqn ① & ②

$\Rightarrow 2[\mu f(x)]^2 - [\delta f(x)]^2 = 4E^{1/2} f(x) \cdot E^{-1/2} f(x)$

$\Rightarrow 4\mu^2 - \delta^2 = 4E^{1/2} \cdot E^{-1/2}$

$\Rightarrow 4\mu^2 - \delta^2 = 4 \quad \Rightarrow \delta^2 = 4\mu^2 - 4$

$\Rightarrow \delta = \sqrt{4\mu^2 - 4}$

$\Rightarrow \delta^2 + 4 = 4\mu^2 \quad \Rightarrow \mu = \sqrt{\frac{\delta^2}{4} + 1}$

$\Rightarrow \Delta \tan^{-1} x = \tan^{-1}(x+h) - \tan^{-1} x$

$\Rightarrow \Delta \tan^{-1} x = \tan^{-1}\left[\frac{x+h-x}{1+(x+h)\cdot x}\right]$

$\Rightarrow \Delta \tan^{-1} x = \tan^{-1} \frac{h}{1+x(x+h)}$

$\Rightarrow f(x) = 1/x \quad , \quad \Delta f(x) = ?$

$\underset{x_0, x_1}{\Delta} f(x) = f(x_0, x_1)$

$= \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{\frac{1}{x_1} - \frac{1}{x_0}}{x_1 - x_0}$

$$\boxed{\underset{x_0, x_1}{\Delta} f(x) = \frac{-1}{x_0 x_1}}$$

$\Rightarrow \underset{x_0, x_1, x_2}{\Delta^2} f(x) = \frac{\underset{x_1, x_2}{\Delta} f(x) - \underset{x_2, x_1}{\Delta} f(x)}{x_2 - x_1}$

$= f(x_1, x_0) - f(x_0 x_1)$

$$\underset{x_0x_1x_2}{\Delta^2} f(x) = \frac{\left(\frac{1}{x_1x_2}\right) - \left(\frac{-1}{x_0x_1}\right)}{x_2 - x_0} = \frac{(-1)^2}{x_0x_1x_2}$$

$$\underset{x_0x_1x_2x_3}{\Delta^3} f(x) = \frac{(-1)^3}{x_0x_1x_2x_3}$$

$$\underset{x_0x_1\ldots x_n}{\Delta^n} f(x) = \frac{(-1)^n}{x_0x_1\ldots x_n}$$

X	Y	Δ	Δ^2	Δ^3
a	$\frac{1}{a}$			
		$\frac{\frac{1}{b}-\frac{1}{a}}{b-a} = \frac{-1}{ab}$		
b	$\frac{1}{b}$		$\frac{\frac{-1}{bc}-\left(\frac{-1}{ab}\right)}{c-a} = \frac{1}{abc}$	
		$\frac{\frac{1}{c}-\frac{1}{b}}{c-b} = \frac{1}{bc}$		$\frac{\frac{1}{bcd}-\frac{1}{abc}}{d-a} = \frac{(-1)^3}{abcd}$
c	$\frac{1}{c}$		$\frac{\frac{-1}{cd}-\frac{1}{bc}}{d-b} = \frac{-1}{bcd}$	
		$\frac{\frac{1}{d}-\frac{1}{c}}{d-c} = \frac{-1}{dc}$		
d	$\frac{1}{d}$			

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Interpolation

Year(x)	1911	1921	1931	1941	1951
Population f(x)	12	17	25	26	35

find $f(15) = ?$, $f(17) = ?$

enterwals $[1911, 1951]$

$y = f(x)$, x = independent variable, equidist

Argument $a, a+h, a+2h, \dots a+\overline{n-1}h, a+nh, \dots$

Entry $= f(a), f(a+h), f(a+2h), \dots f(a+\overline{n-1}h), f(a+nh)$

In $[a, a+nh]$ The values of $f(x)$ is known at some points
let $a+hu$ is any point at which $f(x)$ is to be determined

$$f(a+hu) = E^{u} f(a)$$

$$f(a+hu) = (1+\Delta)^{u} f(a)$$

$$= \left[1 \frac{u}{\angle 1}\Delta + \frac{u(u-1)}{\angle 2}\Delta^2 + \frac{u(u-1)(u-2)}{\angle 3}\Delta^3 + \dots\right] f(a)$$

$$\Rightarrow \boxed{f(a+hu) = f(a) \frac{u}{\angle 1}\Delta f(a) + \frac{u(u-1)}{\angle 2}\Delta^2 f(a) + \dots}$$

Newton Gragory forward enterpolation

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
1911	12				
		5			
1921	17		3		
		8		-10	
1931	25		-7		-25
		1		15	
1941	26		8		
		9			
1951	35				

$\Rightarrow a+hu = 1915 \qquad \Rightarrow 1911 + 10.u = 1915$

$\Rightarrow u = \dfrac{1915 - 1911}{10} \Rightarrow \boxed{u = 0.4}$

from Newton's forward interpolation formula

$$f(1915) = 12 + \frac{0.4}{\angle 1} \times 5 + \frac{0.4(0.4-1)}{\angle 2} \times 3 + \frac{0.4(0.4-1)(0.4-2)}{\angle 3}$$

$$\times(-10) + \frac{0.4(0.4-1)(0.4-2)(0.4-3)}{\angle 4} \times 25$$

$$\Rightarrow f(1915) = 12 + 2 + \frac{0.4(-0.6)}{2} \times 3 + \frac{0.4(-0.6)(-1.6)}{6}$$

$$\times(-10) + \frac{0.4(-0.6)(-1.6)(-2.6)}{24} \times 25$$

$$\Rightarrow f(1915) = 14 + (-0.36) + (-0.64) + (-1.04)$$

$$\Rightarrow f(1915) = 14 - 0.36 - 0.64 - 1.04$$

$$\Rightarrow f(1915) = 11.96$$

Similarly we can calculate f(1917)

★ If our point is close to the last element then

$$f(a+nh+uh) = E^u (a+nh)$$

$$= [(1-\nabla)^{-1}]^u + (a+nh)$$

$$= [1-\nabla]^{-u} + (a+nh)$$

$$= \left[1 + \frac{u}{\angle 1}\nabla + \frac{u(u+1)}{\angle 2}\nabla^2 + \frac{u(u+1)(u+2)}{\angle 3}\nabla^3 + \dots\right] f(a+nh)$$

$$\Rightarrow f(a+nh+uh) = f(a+nh) + \frac{u}{\angle 1}\nabla(a+nh) + \frac{u(u+1)}{\angle 2}\nabla^2$$

$f(a+nh) + \dots$ | Newton Gragory Backward difference

$f(1950) = ?$

$a+nh+uh = 1950$

$1951 + 10u = 1950$

$\Rightarrow u = \frac{1950-1951}{10} = \frac{-1}{10}$

$\Rightarrow \boxed{u = 0.1}$

putting it in formula

$$\Rightarrow f(1950) = 35 + \frac{(-0.1)}{1} \frac{(-0.1)(-0.1+1)}{2} \times 3$$

$$+ \frac{(-0.1)(-0.1+1)(-0.1+2)}{3!} \times 15 +$$

$$\frac{(-0.1)(-0.1+1)(-0.1+2)(-0.1+3)}{4!} \times 25$$

Q1) (0, -1), (1, -1), (2, 1), (4, 11), (5, 19)
find (3, ?)

Soln

X	Y	∇	∇^2	∇^3	∇^4
0	-1				
		$\frac{-1-(-1)}{1-0} = 0$			
1	-1		$\frac{2-0}{2-0} = 1$		
		$\frac{1-(-1)}{2-1} = 2$		0	
2	1		$\frac{5-2}{4-1} = 1$		0
		$\frac{11-1}{4-2} = 5$		0	
4	11		$\frac{8-5}{5-2} = 1$		
		$\frac{19-11}{5-4} = 8$			
5	19				

Newton's divide difference formula

$$f(x, x_0) = \frac{f(x) - f(x_0)}{x - x_0}$$

$$\Rightarrow \boxed{f(x) = f(x_0) + (x - x_0) f(x, x_0)} \quad \text{---} (1)$$

Now

$$f(x, x_0, x_1) = \frac{f(x, x_0) - f(x_0, x_1)}{x - x_1}$$

$$\Rightarrow f(x, x_0) = f(x_0, x_1) + (x - x_1) f(x, x_0 x_1)$$

put this in eqn (1)

$$\Rightarrow f(x) = f(x_0) + (x - x_0)[f(x_0, x_1) + (x - x_1) f(x, x_0 x_1)]$$

$$\Rightarrow \boxed{f(x) = f(x_0) + (x - x_0) f(x_0, x_1) + (x - x_0)(x - x_1) f(x, x_0, x_1)} \quad (2)$$

Now

$$f(x, x_0, x_1, x_2) = \frac{f(x, x_0, x_1) - f(x_0, x_1, x_2)}{x - x_2}$$

$$\Rightarrow f(x, x_0, x_1) = f(x_0, x_1, x_2) + (x - x_2) f(x, x_0, x_1, x_2)$$

put this in eqn (2)

$$\Rightarrow f(x) = f(x_0) + (x - x_0) f(x_0, x_1) f(x - x_0)(x - x_1) f(x_0, x_1, x_2) + (x - x_0)(x - x_1)(x - x_2) f(x_0, x_1, x_2, x_3) + \ldots\ldots$$

$$\Rightarrow f(x) = f(x_0) + (x - x_0) f(x_0, x_1) + (x - x_0)(x - x_1) [f(x_0, x_1, x_2) + (x - x_0) f(x, x_0, x_1, x_2)$$

$$\Rightarrow f(x) = f(x_0) + (x - x_0) f(x_0, x_1) + (x - x_0)(x - x_1) f(x_0, x_1, x_2) + (x - x_0)(x - x_1)(x - x_2) f(x, x_0, x_1, x_2)$$

Similarly,

$$\Rightarrow f(x) = f(x_0) + (x - x_0) f(x_0, x_1) + (x - x_0)(x - x_1)$$

$f(x_0,x_1,x_2)+\ldots(x-x_0)(x-x_1)\ldots(x-x_{n-1})f(x,x_0,\ldots)+R_n(x)$

where $R_n(x)=(x-x_0)(x-x_1)(x-x_2)\ldots(x-x_n)f(x,x_0\ldots)$

$\Rightarrow f(x)=-1+(x-0)\times 0+(x-0)(x-1)\times 1+(x-0)(x-1)(x-2)\times 0+0$

$\Rightarrow f(x)=-1+0+x(x-1) \Rightarrow f(x)=x^2-x-1$

$\Rightarrow f(3)=3^2-3-1=5$ //

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The Human Cost of the israel-palestinian conflict (Death/enjuries)

Year	Palestine	Israel
2008	3302	853
2009	7460	123
2010	1659	185
2011	2260	136
2012	4936	578
2013	4031	157
2014	19860	2796
2015	14813	339
2016	3572	222
2017	8526	174
2018	31558	130
2019	15628	133
2020	2781	61

eqn of palestine = a_1+b_1 year

eqn of Israel = a_2+b_2 year

Solve this and do predictive analysis

Q)

x	-1	1	3	5	7
Y	-4	-4	4	20	44

find $f(0), f(4), f(6), f(8)$

Soln first construct difference table

X	$f(x)$	Δ	Δ^2	Δ^3	Δ^4
-1	-4				
		0			
1	-4		8		
		8		0	
3	4		8		0
		16		0	
5	20		8		
		24			
7	44				

$$f(a+hu) = f(a) + \frac{u}{\angle 1}\Delta f(a) + \frac{u(u-1)}{\angle 2}\Delta^2 f(a)$$

let $a+hu = 0$

$-1+2.u = 0 \Rightarrow \boxed{u = 1/2}$

$$f(0) = -4 + 0.5\times 0 + \frac{0.5(0.5-1)}{\cancel{2}\,1}\times \cancel{8}\,4$$

$f(0) = -4 + 0 + 0.5(-0.5)\times 4$

$f(0) = -4 + (-1)$

$f(0) = 5$

let $a+hu = x$

$$= -1 + 2.u = x$$

$$\boxed{u = x + 1/2}$$

$$f(x) = -4 + \left(\frac{x+1}{2}\right) \times 0 + \frac{\left(\frac{x+1}{\cancel{2}}\right)\left(\frac{x+1}{2} - 1\right)}{\cancel{2}} \times \cancel{8}^{2}$$

$$f(x) = -4 + 0 + 2\left(\frac{x-1}{2}\right)(x+1)$$

$$f(x) = -4 + x^2 - 1$$

$$\boxed{f(x) = x^2 - 5} \rightarrow \text{General}$$

↳ Let a th $u = 6$

$\Rightarrow -1 + 2.u = 6 \Rightarrow \boxed{u = 3.5}$

↳ Let a th $u = 8$

$-1 + 2u = 8$

$u = \frac{8+1}{2} \rightarrow \boxed{u = 4.5}$

Now when the difference is not symmetric

	X	Y			
$x_0 \rightarrow$	-1	-4			
			$\frac{4+4}{3-(-1)} = 2$		
				$\frac{8-2}{5-(-1)} = 1$	
$x_1 \rightarrow$	3	4			
			$\frac{20-4}{5-3} = 8$		0
				$\frac{12-8}{7-3} = 1$	
$x_2 \rightarrow$	5	20			
			$\frac{44-20}{7-5} = 12$		
$x_3 \rightarrow$	7	44			

$$\Rightarrow f(x) = f(x_0) + (x - x_0)\Delta f(x) + (x - x_0)(x - x_1)\Delta f(x)$$
$$+ (x - x_0)(x - x_1)(x_1 - x_2)\Delta^3 f(x)$$

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$\Rightarrow f(x) = -4 + (x+1)x_2 + (x+1)(x-3)x_1 + 0$

$\Rightarrow f(x) = -4 + 2x + 2 + x^2 - 2x - 3$

$\Rightarrow \boxed{f(x) = x^2 - 5}$

Lagrangee's Interpolation

x	x_0	x_1	x_2	x_3	...	x_n
y	y_0	y_1	y_2	y_3	...	y_n

$$y(x) = \frac{(x-x_1)(x-x_2)(x-x_3)\dots(x-x_n)}{(x_0-x_1)(x_0-y_2)(x_0-y_3)\dots(x_0-y_n)} \times y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)\dots(x-x_n)}{(x_1-y_0)(x_1-x_2)(x_1-x_3)\dots(x_1-y_n)} \times y_1 + \dots\dots + \frac{(x-x_0)(x-x_1)(x-x_2)\dots(x-x_{n-1})}{(x_n-y_0)(x_n-y_1)(x_n-y_2)\dots(x_n-y_{n-1})} \times$$

$\rightarrow f = x^2 - 5$

x	-1	1	3	5	7
y	-4	-4	4	20	44

$$y(x) = \frac{(x-1)(x-3)(x-5)(x-7)}{(-1-1)(-1-3)(-1-5)(-1-7)} \times (-4) + \frac{(x+1)(x-3)(x-5)(x-7)}{(1+1)(1-3)(1-5)(1-7)} \times (-4) + \frac{(x+1)(x-1)(x-5)(x-7)}{(3+1)(3-1)(3-5)(3-7)} \times (4) + \frac{(x+1)(x-1)(x-3)(x-7)}{(5+1)(5-1)(5-3)(5-7)} \times 20 +$$

$$\frac{(x+1)(x-1)(x-3)(x-5)}{(7+1)(7-1)(7-3)(7-5)} \times 44$$

after solving this we get $Y(x) = x^2 - 5$

From lagrange's Interpolation

$$Y(x) = L_0(x) * Y_0 + L_1(x) y_1 + \dots L_n(x) y_n$$

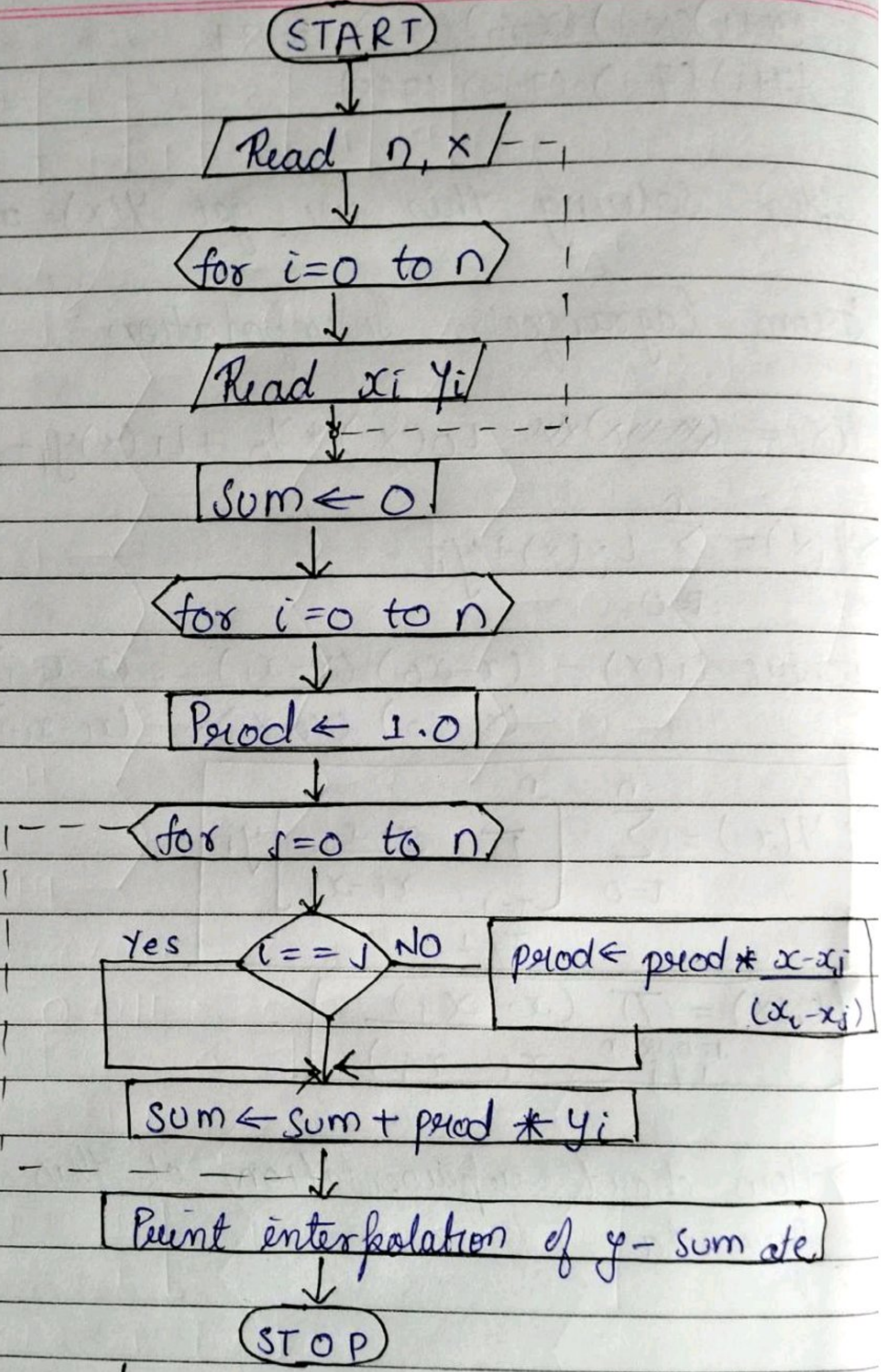
$$Y(x) = \sum_{i=0}^{n} L_i(x) + y_i$$

where $L_i(x) = \frac{(x-x_0)(x-x_1)\dots(x-x_{i-1})\dots(x-x_n)}{(x_i-x_0)(x_i-x_1)\dots(x_i-x_{i+1})\dots(x_i-x_n)}$

$$\boxed{Y(x) = \sum_{i=0}^{n} \left[\prod_{\substack{j=1 \\ j \neq i}}^{n} \frac{x-x_j}{x_i-x_j} \right] y_i}$$

$$\boxed{L_i(x) = \prod_{\substack{j=0,1,2\dots n \\ i \neq j}} \frac{(x-x_j)}{x_i-x_j)}}$$

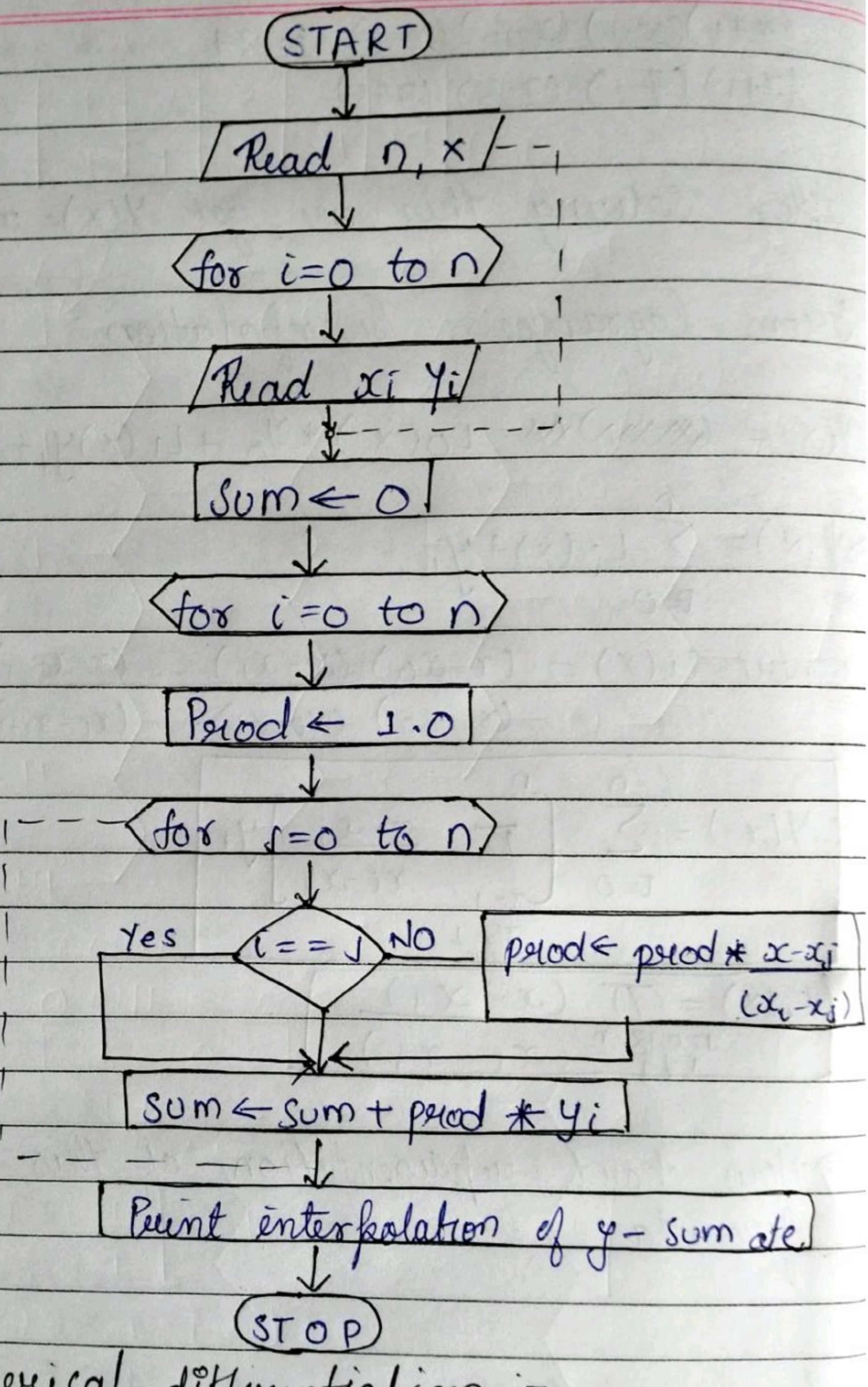
⇒ flow chart representation of this formula



Numerical differentiation :-

$y = f(x)$

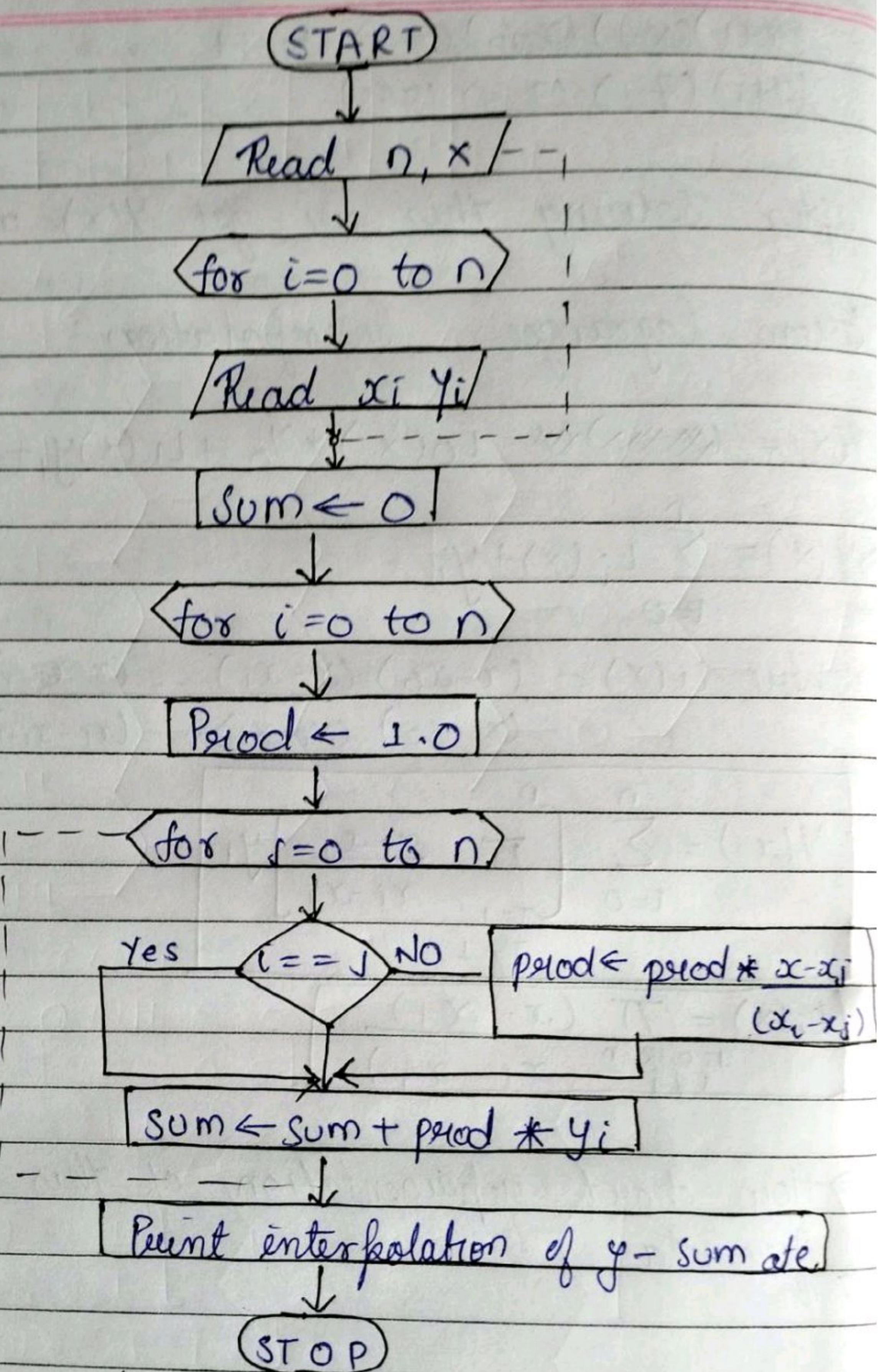
$x \rightarrow$	a	$a+h$	$a+2h$	...	$a+nh$
$y \rightarrow$	y_0	y_1	y_2	...	y_n



Numerical differentiation :-

$y = f(x)$

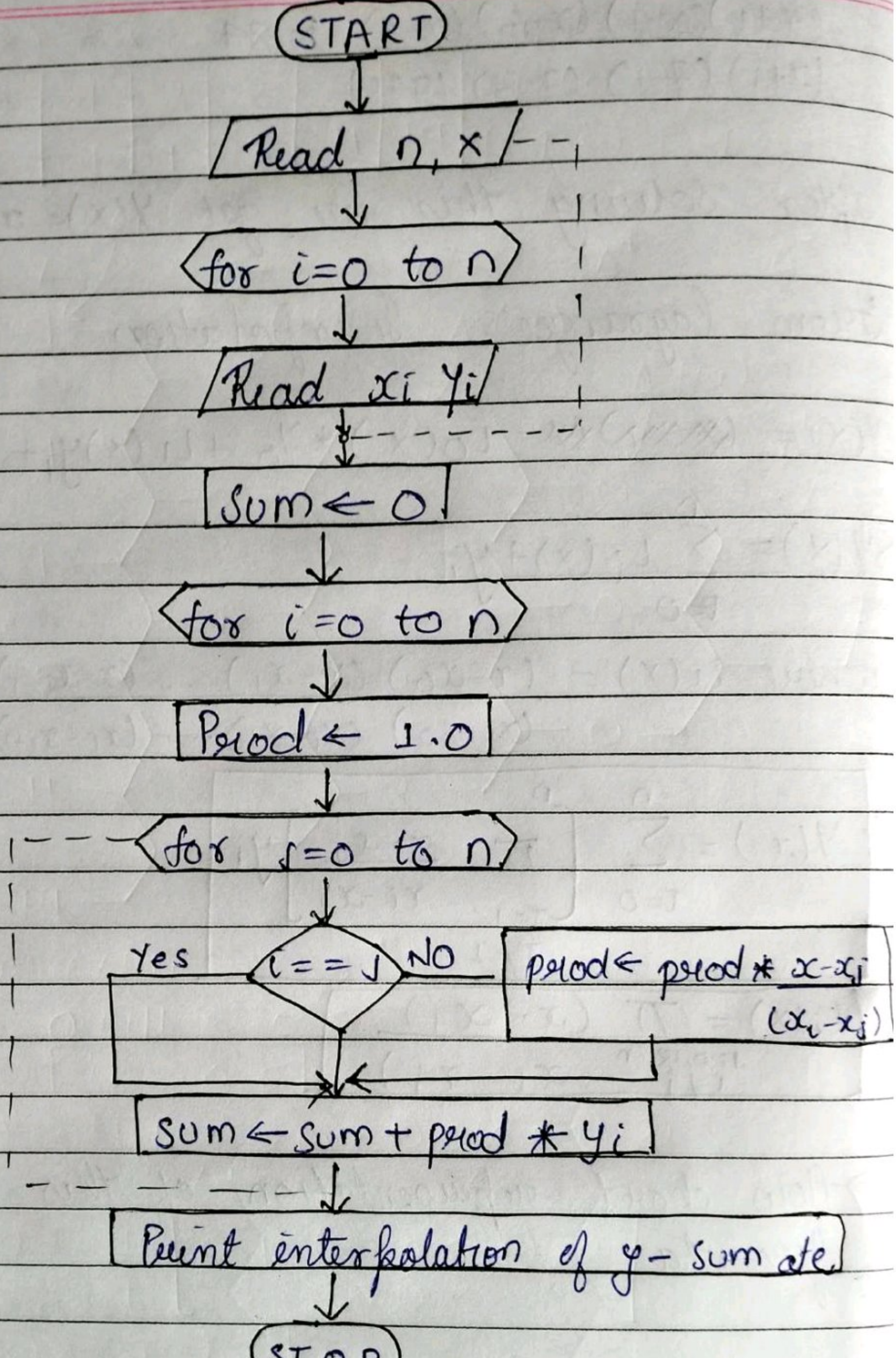
$x \rightarrow$	a	$a+h$	$a+2h$	...	$a+nh$
$Y \rightarrow$	Y_0	Y_1	Y_2	...	Y_n



Numerical differentiation :-

$y = f(x)$

$x \rightarrow$	a	$a+h$	$a+2h$	..	$a+nh$
$Y \rightarrow$	y_0	y_1	y_2	...	y_n



Numerical differentiation :-

$y = f(x)$

$a, a+h, a+2h \ldots a+nh$

$f(a), f(a+h), f(a+2h) \ldots f(a+nh)$

$$\Rightarrow f(a+hu) = f(a) + \frac{u}{\lfloor 1} \Delta f(a) + \frac{u(u-1)}{\lfloor 2} \Delta^2 f(a)$$

$$+ \frac{u(u-1)(u-2)}{\lfloor 3} \Delta^3 f(a) \ldots$$

on differentiating with r to u

$$\Rightarrow h f'(a+hu) = 0 + \Delta f(a) + \frac{(2u-1)}{2} \Delta^2 f(a)$$

$$+ \frac{(3u^2 - 6u + 2)}{6} \Delta^3 f(a) + \ldots$$

again differentiate w. r to u

$$\Rightarrow h^2 f''(a+hu) = 0 + \cancel{\Delta f(a)}^{0} + \cancel{\frac{(2u-1)}{2}} + \Delta^2 f(a)$$

$$+ \frac{(6u-6)}{6} \Delta^3 f(a)$$

again diff. w. r to u

$$\Rightarrow h^3 f'''(a+hu) = 0 + 0 + 0 + \Delta^3 f(a) + \ldots$$

$$\Rightarrow f'(a+hu) = \frac{1}{h} \left[\Delta f(a) + \frac{(2u-1)}{2} \Delta^2 f(a) + \frac{(3u^2 - 6u + 2)}{6} \Delta^3 f(a) + \ldots \right]$$

$$\Rightarrow f''(a+hu) = \frac{1}{h^2} \left[\Delta^2 f(a) + (u-1) \Delta^3 f(a) + \ldots \right]$$

$$\Rightarrow f'''(a+hu) = \frac{1}{h^3} \left[\Delta^3 f(a) + \ldots \right]$$

Q) $f(x) = x^2 - x - 1$ find $f(0.5), f'(0.5)$, $f''(0.5), f'''(0.5)$, $f(4.5), f'(4.5), f''(4.5)$, $f'''(4.5)$

0	1	2	3	4	5
-1	-1	1	5	11	19

x	Y ($f(a)$)	Δ^2 ($\Delta f(a)$)	Δ^2 ($\Delta^2 f(a)$)	Δ^4 ($\Delta^3 f(a)$)	Δ^5 ($\Delta^4 f(a)$)	Δ^6 ($\Delta^5 f$)
0	-1					
		0				
			2			
1	-1			0		
		2			0	
2	1		2			
		4		0		
3	5		2		0	
		6		0		
4	11		2			
		8				
5	19					

now $a + hu = 0.5$

$a + 1u = 0.5$

$\boxed{u = 0.5}$

$f(0.5) = -1 + \frac{0.5}{\lfloor 1} \times 0 + \frac{0.5(0.5-1)}{\lfloor 2} \times 2$

$\Rightarrow f'(0.5) = \frac{1}{1}\left[0 + \frac{(2 \times 0.5 - 1) \times 2}{2}\right]$

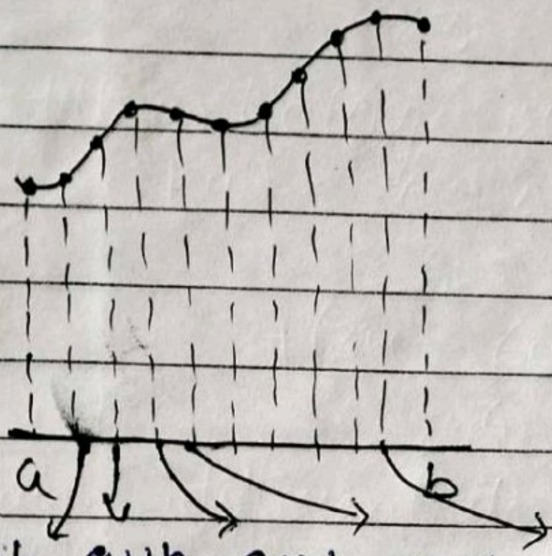
$\Rightarrow f''(0.5) = \frac{1}{1^2}[2]$

$\Rightarrow f'''(0.5)\ \frac{1}{1^3}[0]$

Newton cote's formula

$I = \int_a^b f(x)\,dx$

Integration is finding the area under curve $f(x)$ and x axis within the range a to b



$a+h \quad a+2h \quad a+3h \quad a+4h \quad a+nh$

$\Rightarrow I : \underbrace{\int_a^b f(x)}_{\text{given function}} \simeq \underbrace{\int_a^{a+nh} y\,dx}_{\text{derived polynomial}}$

$\Rightarrow I = \int_a^{a+nh} [L_0(x)(Y_0) + L_1(x)Y_1 + \dots L_n(x)Y_n]\,dx$

$\Rightarrow I = \int_a^{a+nh} L_0(x)Y_0\,dx + \int_a^{a+nh} L_1(x)Y_1\,dx + \dots \int_a^{a+nh} L_n(x)Y_1\,dx$

$\Rightarrow I = \sum_{i=0}^{n} y_i \int_a^{a+nh} L_i(x)\,dx$

put $x = a+hu$

$dx = h\,du$

$\Rightarrow$ At $x = a$	At $x = a+nh$
$a = a+hu$	$a+nh = a+hu$
$u = 0$	$n = u$

$$\boxed{I = n \sum_{i=0}^{n} y_i c_i} \quad \boxed{\frac{1}{n} \int_0^n L_i(u)\, du} \text{ cotes no.}$$

$$\Rightarrow \boxed{I = nh \sum_{i=0}^{n} y_i c_i^n} \quad \text{where } c_i^n = \frac{1}{n} \int_0^n L_i(u)\,du$$

Newton cotes formula

⇒ Derivation Trapezoidal Rule from Newton cote's formula

we know that

$$I = nh \sum_{i=0}^{n} y_i {}^n c_i \quad \text{where } c_i^n = \frac{1}{n} \int_0^n L_i(u)\,du \quad \text{— (1)}$$

put $n = 1$

$$I = 1h \sum_{i=0}^{1} y_i c_i^1$$

$$\Rightarrow I = h[y_0 c_0^1 + y_1 c_1^1]$$

$$\Rightarrow I = h\left[y_0 \frac{1}{2} + y_1 \frac{1}{2}\right]$$

$$\Rightarrow I = \frac{h}{2}[y_0 + y_1]$$

$$\Rightarrow I_1 = \frac{h}{2}[y_0 + y_1]$$

$$\Rightarrow I_2 = \frac{h}{2}[y_1 + y_2]$$

$$\vdots$$

$$\Rightarrow I_n = \frac{h}{2}[y_{n-1} + y_n]$$

put $n = 1$, $i = 0$

$$c_0^1 = \frac{1}{1} \int_0^1 L_0(u)\,du$$

$$c_0^1 = \int_0^1 \frac{u-1}{0-1}\,du$$

$$c_0^1 = \int_0^1 (1-u)\,du$$

$$c_0^1 = \left[u - \frac{u^2}{2}\right]_0^1$$

$$c_0^1 = 1 - \frac{1}{2}$$

$$\Rightarrow \boxed{c_0^1 = 1/2}$$

put $n = 1$, $i = 1$

$$c_1^1 = \frac{1}{1} \int_0^1 L_1(u)\,du$$

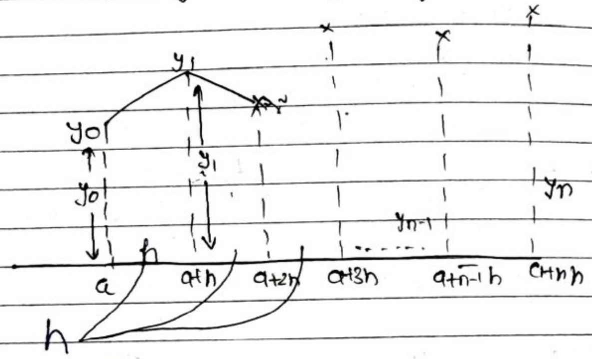
$$c_1^1 = \int_0^1 \left(\frac{u-0}{1-0}\right) du$$

$$c_1^1 = \left[\frac{u^2}{2}\right]_0^1 \quad \boxed{c_1^1 = 1/2}$$

⇒ $I_1 + I_2 \dots + I_n = \frac{h}{2}\left[(y_0+y_1) + 2(y_1+y_2+\dots y_{n-1})\right]$

⇒ $I = \frac{h}{2}\left[(y_0+y_1) + 2(y_1+y_2+\dots y_{n-1})\right]$

X	0	1	2	3	4	5
f(x)	-1	-1	1	5	11	19
	y_0	y_1	y_2	y_3	y_4	y_5



$I = \int_0^5 (x^2-x-1)\,dx$

$= \left[\frac{x^3}{3} - \frac{x^2}{2} - x\right]_0^5 = \frac{25^3}{3} - \frac{25^2}{2} - 5$

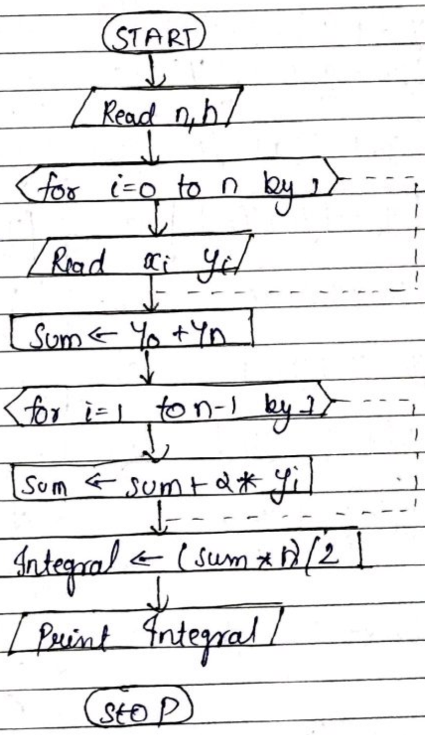
⇒ $I = \int_0^5 f(x)\,dx$

⇒ $I = \int_0^5 f(x) = \frac{h}{2}\left[(y_0+y_5) + 2(y_1+y_2+y_3+y_4)\right]$

$I = \frac{1}{2}\left[(-1+19) + 2(-1+1+5+11)\right]$

$= \frac{1}{2}\left[18 + 2\times16\right] = \frac{1}{2}\times 50$

$I = 25$ sq. unit



Trapezoidal Rule flowchart

⇒ Derivation Simpson's 1/3 rule from NCF

we know that

$I = nh \sum_{i=0}^{n} y_i c_i^n$

put $n=2$

$I = 2h \sum_{i=0}^{2} y_i c_i^2$

$I = 2h [y_0 c_0^2 + y_1 c_1^2 + y_2 c_2^2]$

$I = 2h \left[\frac{1}{6} y_0 + \frac{4}{6} y_1 + \frac{1}{6} y_2\right]$

$I = \frac{2h}{6} [y_0 + 4y_1 + y_2]$

$$\boxed{I = \frac{h}{3} [y_0 + 4y_1 + y_2]}$$

↳ Simpson's 1/3 rule

$I_1 = \frac{h}{3} [y_0 + y_1 + y_2]$

$I_2 = \frac{h}{3} [y_2 + y_3 + y_4]$

$I_3 = \frac{h}{3} [y_4 + y_5 + y_6]$

⋮

$I_n = \frac{h}{3} [y_{n-2} + 4y_{n-1} + y_n]$

$I_1 + I_2 \dots I_n = \frac{h}{3} [(y_0 + y_1) + (y_1 + y_3 + y_5 \dots + y_{n-1}) + 2(y_2 + y_4 + y_6 + \dots + y_{n-2})]$

where $c_i^n = \frac{1}{n} \int_0^n L_i(u)\,du$

put $n=2$, $i=0$

$c_0^2 = \frac{1}{2} \int_0^2 L_0(u)\,du$

$c_0^2 = \frac{1}{2} \int_0^2 \frac{(u-1)(u-2)}{(0-1)(0-2)}\,du$

$c_0^2 = \frac{1}{4} \int_0^2 [u^2 - 3u + 2]\,du$

$c_0^2 = \frac{1}{4} \left[\frac{u^3}{3} - \frac{3u^2}{2} + 2u\right]_0^2$

$c_0^2 = \frac{1}{4} \left[\frac{8}{3} - \frac{12}{2} + 4\right]$

$c_0^2 = \frac{1}{4} \left[\frac{8}{3} - 2\right] \Rightarrow \boxed{c_0^2 = \frac{1}{6}}$

Put $n=2$, $i=1$

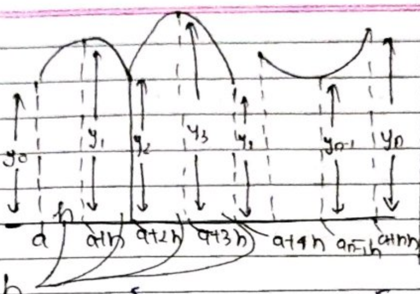
$c_1^2 = \frac{1}{2} \int_0^2 L_1(u)\,du$

$c_1^2 = \frac{1}{2} \int_0^2 \frac{u(u-2)}{1(1-2)}\,du$

$c_1^2 = \frac{1}{2} \int_0^2 u(2-u)\,du$

$c_1^2 = \frac{1}{2} \left[u^2 - \frac{u^3}{3}\right]_0^2$

$c_1^2 = \frac{1}{2} \left[4 - \frac{8}{3}\right] \rightarrow \boxed{c_1^2 = \frac{4}{6}}$



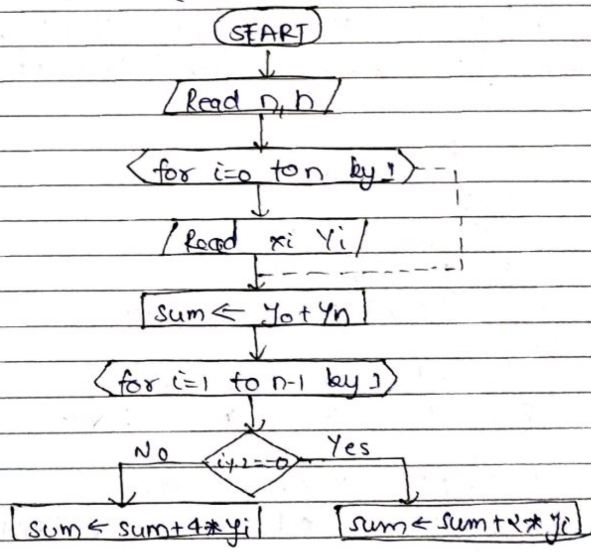
Now $c_i^n = c_{n-i}^n$

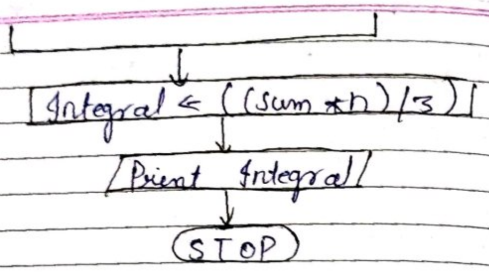
put $n=2$, $i=2$

$$\boxed{c_2^2 = c_0^2 = \frac{1}{6}}$$

$I = \int_0^5 (x^2 - x - 1)\,dx = \left[\frac{x^3}{3} - \frac{x^2}{2} - x\right]_0^5$

$I = \frac{25^3}{3} - \frac{25^2}{2} - 5$





⇒ Derivation Simpson's 3/8 rule from NCF

put $n = 3$

$I = 3h \sum_{i=0}^{3} y_i C_i^3 \qquad i = 0, 1, 2, 3$

$I = 3h\left[y_0 C_0^3 + y_1 C_1^3 + y_2 C_2^3 + y_3 C_3^3\right]$ — (1)

from Cote's number we'll calculate $C_0^3, C_1^3, C_2^3, C_3^3$

$C_0^3 = \frac{1}{3}\int_0^3 L_0(u)\,du$

$C_0^3 = \frac{1}{3}\int_0^3 \frac{(u-1)(u-2)(u-3)}{(0-1)(0-2)(0-3)}\,du$

$C_0^3 = \frac{1}{18}\int_0^3 \left[(1-u)(u^2-5u+6)\right]du$

$C_0^3 = \frac{1}{18}\left[-\frac{u^4}{4} + \frac{6u^3}{3} - \frac{11u^2}{2} + 6u\right]_0^3$

$C_0^3 = \frac{1}{18}\left[-\frac{81}{4} + \frac{72\times4}{84} - \frac{99\times2}{2\times2}\right] = \frac{1}{2\cdot 18\times 4}$

$\boxed{C_0^3 = 1/8}$

$\Rightarrow C_1^3 = \frac{1}{3}\int_0^3 L_1(u)\,du$

$\Rightarrow C_1^3 = \frac{1}{3}\int_0^3 \frac{u(u-2)(u-3)}{1(1-2)(1-3)}\,du$

$\Rightarrow C_1^3 = \frac{1}{6}\left[\frac{u^4}{4} - \frac{5u^3}{3} + 3u^2\right]_0^3$

$= \frac{1}{6}\left[\frac{81}{4} - 18\right] = \frac{81-72}{6\times4}$

$\Rightarrow C_1^3 = \frac{\cancel{9}\,3}{\cancel{24}\,8} \qquad \boxed{C_1^3 = \frac{3}{8}}$

$\Rightarrow C_2^3 = C_{3-2}^3 \Rightarrow \boxed{C_2^3 = C_1^3 = 3/8}$

$\Rightarrow C_3^3 = C_{3-3}^3 \Rightarrow \boxed{C_3^3 = C_0^3 = 1/8}$

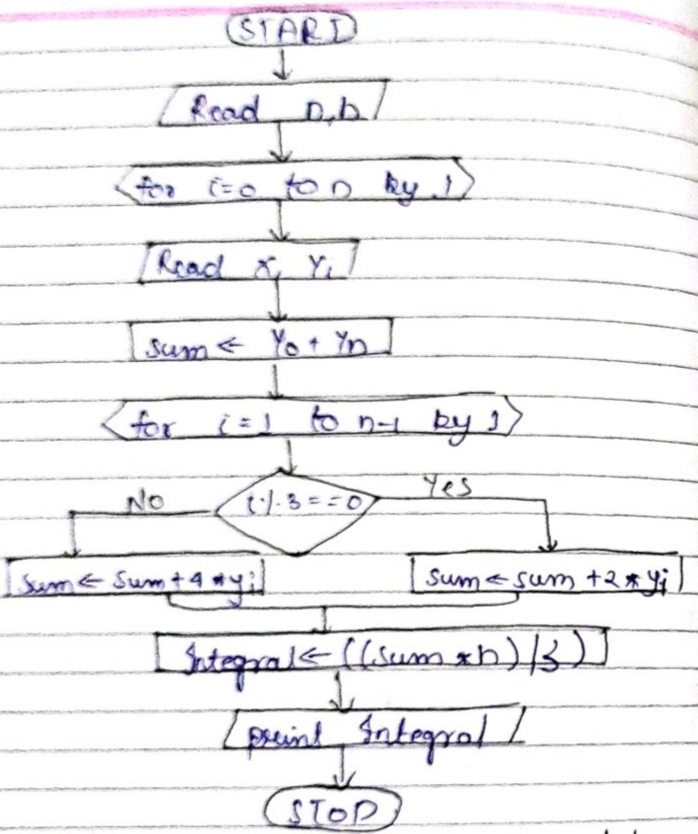
$I_1 = \frac{3h}{8}\left[y_0 + 3y_1 + 3y_2 + y_3\right]$

$I_2 = \frac{3h}{8}\left[y_3 + 3y_4 + 3y_5 + 3y_6\right]$

$\vdots$

$I_n = \frac{3h}{8}\left[y_{n-3} + 3y_{n-2} + 3y_{n-1} + 3y_n\right]$

$\Rightarrow I = \frac{3h}{8}\left[y_0 + y_n + 3(y_1 + y_2 + \dots y_{n-1}) + 2(y_3 + y_6 + \dots y_{n-3})\right]$



26/10/2023

Numerical Integration

$$I = \int_a^b f(x)\,dx$$

$x = a + hu$ at $x = a$, $u = 0$

$dx = h\,du$ $\quad x = b$, $u = n$

where ; $b = a + nh$

$$\Rightarrow I = h \int_0^n f(a + hu)\,du$$

$$\Rightarrow I = h \int_0^n \left[f(a) + \frac{u}{\angle 1} \Delta f(a) + \frac{u(u-1)}{\angle 2} \Delta^2 f(a) \dots \right] du$$

$$\Rightarrow I = h \left[f(a)\,u + \frac{u^2}{2} \Delta f(a) + \dots \right]_0^n$$

$$\Rightarrow \boxed{I = nh \left[f(a) + \frac{n}{2} \Delta f(a) + \left(\frac{n^2}{3} - \frac{n}{2} \right) \frac{\Delta^2 f(a)}{2} \dots \right]}$$

↳ General quadrature formula

put $n = 1$, reducing higher power

$$I = 1 \cdot h \left[f(a) + \frac{1}{2} \Delta f(a) \right]$$

$$I = h \left[f(a) + \frac{f(a+h) - f(a)}{2} \right]$$

$$\boxed{I = \frac{h}{2} \left[f(a) + f(a+h) \right]} \Rightarrow I = \frac{h}{2} [y_0 + y_1]$$

↳ Trapezoidal rule

put $n = 2$, reducing higher power

$$I = 2h \left[f(a) + \frac{2}{2} \Delta f(a) + \left(\frac{2^2}{3} - \frac{2}{2} \right) \frac{\Delta^2 f(a)}{2} \right]$$

$$I = 2h \left[\cancel{f(a)} + f(a+h) - \cancel{f(a)} + \left(\frac{2}{3} - \frac{1}{2} \right) \left(f(a+2h) - 2f(a+h) + f(a) \right) \right]$$

$$I = \frac{2h}{6} \left[6f(a+h) + f(a+2h) - 2f(a+h) + f(a) \right]$$

$$I = \frac{h}{3} \left[f(a) + 4f(a+h) + f(a+2h) \right]$$

$$I = \frac{h}{3}\left[Y_0 + 4Y_1 + Y_2\right] \rightarrow \text{Simpson's } \frac{1}{3} \text{ rule}$$

Q) $\int_0^1 \frac{1}{1+x^2}\,dx$

$a = 0,\ b = 1,\ f(x) = \frac{1}{1+x^2}$

$n = 6 \Rightarrow h = \frac{b-a}{n} \Rightarrow \frac{1-0}{6}$

$\Rightarrow \boxed{h = 1/6}$

i	x	$y = \frac{1}{1+x^2}$
0	0	$\frac{1}{1+0^2} = 1$
1	$\frac{1}{6}$	$\frac{1}{1+(1/6)^2} = \frac{36}{37}$
2	$\frac{2}{6}$	$\frac{1}{1+(2/6)^2} = \frac{36}{40}$
3	$\frac{3}{6}$	$\frac{1}{1+(3/6)^2} = \frac{36}{45}$
4	$\frac{4}{6}$	$\frac{1}{1+(4/6)^2} = \frac{36}{52}$
5	$\frac{5}{6}$	$\frac{1}{(1+5/6)^2} = \frac{36}{61}$
6	$\frac{6}{6}$	$\frac{1}{(1+1)^2} = \frac{1}{2}$

	I_1	I_2	I_3	I_4	I_5	I_6
x	0	1/6	2/6	3/6	4/6	5/6	6/6
y	1	36/37	36/40	36/45	36/52	36/61	1/2

By Trapezoidal rule

$$\int_0^1 \frac{1}{1+x^2}\,dx = \frac{h}{2}\left[(Y_0+Y_6) + 2(Y_1+Y_2+Y_3+Y_4+Y_5)\right]$$

$$= \frac{1/6}{2}\left[\left(1+\frac{1}{2}\right) + 2\left(\frac{36}{37} + \frac{36}{40} + \frac{36}{45} + \frac{36}{52} + \frac{36}{61}\right)\right]$$

$\Rightarrow I_1 =$ $\boxed{E_T = I - I_1}$

Similarly we can do it with Simpson's 1/3

$$I_2 = \frac{h}{3}\left[(Y_0+Y_6) + 4(Y_1+Y_3+Y_5) + 2(Y_2+Y_4)\right]$$

$$= \frac{1/6}{3}\left[\left(1+\frac{1}{2}\right) + 4\left[\frac{36}{37} + \frac{36}{45} + \frac{36}{61}\right] + 2\left[\frac{36}{40} + \frac{36}{52}\right]\right]$$

$\Rightarrow \boxed{E_{S_{1/3}} = I - I_2}$

By Simpson's 3/8 rule

$$\int_0^1 \frac{1}{1+x^2}\,dx = \frac{3h}{8}\left[(Y_0+Y_6) + 3(Y_1+Y_2+Y_4+Y_5) + 2(Y_3)\right]$$

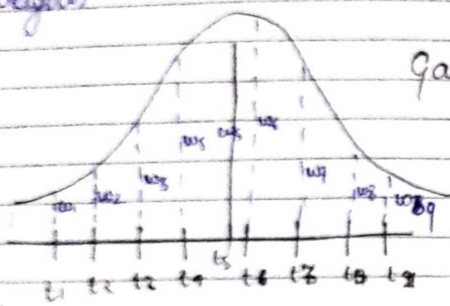
$$I_3 = \frac{3\times 1/6}{8}\left[\left(1+\frac{1}{2}\right) + 3\left(\frac{36}{37} + \frac{36}{40} + \frac{36}{52} + \frac{36}{61}\right)\right.$$

$+ 2 \times \frac{56}{45} \Big]$

$\Rightarrow I_3 =$

$$\boxed{E_{S_{3/8}} = I - I_3}$$

ω = weights



Gaussian curve or Bell curve

$\Rightarrow \int_{-1}^{+1} f(x)dx = \omega_1 f(t_1) + \omega_2 f(t_2) + \omega_3 f(t_3) + \dots + \omega_4 f(t_4)$

$\Rightarrow \sum_{i=1}^{n} \omega_i f(t_i)$

NOTE :- we can do this only when the range is from -1 to $+1$

When we only want to use this formula then

eg :- $\int_a^b f(x)dx$

put $x = \frac{b-a}{2}t + \left(\frac{b+a}{2}\right)$

$\Rightarrow dx = \left(\frac{b-a}{2}\right)t$

at $x = a$ $\quad t = -1$

$x = b$ $\quad t = +1$

$a = \frac{b-a}{2}t + \left(\frac{b+a}{2}\right)$

$a - \left(\frac{b+a}{2}\right) = \frac{b-a}{2}t$

$\frac{2a-b-a}{2} = \frac{b-a}{2}t$

$\Rightarrow \boxed{t = -1}$

$b = \frac{b-a}{2}t + \left(\frac{b+a}{2}\right)$

$b - \left(\frac{b+a}{2}\right) = \left(\frac{b-a}{2}\right)t$

$\frac{2b-b-a}{2} = \left(\frac{b-a}{2}\right)t$

$\boxed{t = +1}$

$\Rightarrow$ Program

```
#include <stdio.h>
#include <conio.h>
#include <math.h>

float f (float x)
{ return (1/(1+x*x)); }
void main()
{ float a, b, h, integral, sum;
  int i, n;
  clrscr();
  printf("\n Enter range a,b and n\n");
  scanf("%f %f %d", &a, &b, &n);
  sum = f(a) + f(b) / 2 ;
```

```
for (i = 1; i<n; i++)
    sum = sum + f(a+i*h)
integral = h * sum
printf("\n Integral using trapezoid rule = %f", integral);
getch();
}
```

* we can change the code of for loop with:-

(1) `(i%2==0)? sum+=4*f(a+ih): sum += 2*f(a+ih)`

integral = $\frac{h}{3}$ * sum

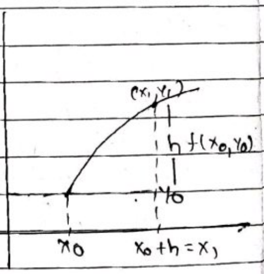
(2) `(i%3==0)? sum+= 3*f(a+ih) : sum+=2*f(a+ih)`

integral = $\frac{3h}{8}$ * sum

Solving a differential eqn 07/10/23

$\frac{dy}{dx} = f(x,y); \quad y(x_0) = y_0$

$\left.\frac{dy}{dx}\right|_{x_0,y_0} = f(x_0,y_0)$



$\Rightarrow \frac{y_1 - y_0}{x_1 - x_0} = f(x_0,y_0)$

$\Rightarrow y_1 - y_0 = h(x_0,y_0)$

$\Rightarrow \boxed{y_1 = y_0 + h f(x_0,y_0)}$

$\boxed{y_2 = y_1 + h f(x_1,y_1)}$

$\Rightarrow \boxed{y_{n+1} = y_n + h f(x_n, y_n)}$ → General formula

ex:- $\frac{dy}{dx} = x+y$; $y(0) = 1$, $y(1) = ?$ in steps of 0.25.

Given $f(x,y) = x+y$

$x_0 = 0$

$y_0 = 1$, $h = 0.25$

x_0	x_1	x_2	x_3	x_4
0	0.25	0.50	0.75	1.0
1	1.25			

y_0 — To be determined

$\Rightarrow y_1 = y_0 + h f(x_0,y_0)$

$\Rightarrow y_1 = y_0 + h[x_0 + y_0]$

$\Rightarrow y_1 = 1 + 0.25[0+1] \Rightarrow \boxed{y_1 = 1.25}$

Now,

$\Rightarrow y_2 = y_1 + h f(x_1,y_1)$

$\Rightarrow y_2 = y_1 + h[x_1+y_1]$

$\Rightarrow y_2 = 1.25 + 0.25[0.25 + 1.25] \Rightarrow \cancel{y_2}$

$\Rightarrow y_2 = 1.25 + 0.25 \times 1.5 \Rightarrow \boxed{y_2 = 1.625}$

Now,

$\Rightarrow y_3 = y_2 + h f(x_2, y_2)$

$\Rightarrow y_3 = y_2 + h[x_2 + y_2]$

$\Rightarrow y_3 = 1.625 + 0.25[0.50 + 1.625]$

$\Rightarrow y_3 = 1.625 + 0.25 \times 2.125$

$\Rightarrow y_3 = 1.625 + 0.53125 \Rightarrow \boxed{y_3 = 2.15625}$

This method is called Euler's method
This method is not correct as it does not give the correct value. It gives the approx value not the exact value.
To correct this we have a formula named Correction formula :-

$$\boxed{y_n^{(i+1)} = y_{n-1} + \frac{h}{2}\left[f(x_{n-1}, y_{n-1}) + f(x_n, y_n^{(i)})\right]}$$

put $n=1$, $i=0$

$\Rightarrow y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$

$\Rightarrow y_1^{(1)} = 1 + \frac{0.25}{2}\left[(0+1) + (0.25+1.25)\right]$

$\Rightarrow y_1^{(1)} = 1 + \frac{0.25}{2}[2.5]$

$\Rightarrow y_1^{(1)} = 1 + \frac{0.625}{2}$ $\Rightarrow \boxed{y_1^{(1)} = 1.3125}$

put $n=1$, $i=1$ in Corrector formula

$\Rightarrow y_1^{(2)} = 1 + \frac{0.25}{2}\left[(0+1) + (0.25+1.3125)\right]$

$\Rightarrow y_1^{(2)} = 1 + \frac{0.25}{2} \times 2.5625$

$\Rightarrow y_1^{(2)} = 1 + \frac{0.640625}{2}$ $\Rightarrow y_1^{(2)} = 1 + 0.3203125$

$\Rightarrow \boxed{y_1^{(2)} = 1.320325}$

put $n=1$, $i=2$ in Corrector formula

$\Rightarrow y_1^{(3)} = 1 + \frac{0.25}{2}\left[(0+1) + (0.25+1.320325)\right]$

$\Rightarrow \boxed{y_1^{(3)} = 1.321289}$

put $n=1$, $i=3$ in Corrector formula

$\Rightarrow y_1^{(4)} = 1 + \frac{0.25}{2}\left[(0+1) + (0.25+1.321289)\right]$

$\Rightarrow y_1^{(4)} = 1 + \frac{0.25}{2} \times 2.571289$

$\Rightarrow y_1^{(4)} = 1 + \frac{0.642822}{2}$

$\Rightarrow y_1^{(4)} = 1 + 0.3214111$

$\Rightarrow \boxed{y_1^{(4)} = 1.3214111}$

★ Continue this until we got the value same as previous one.

Q1) $\frac{dx}{dy} = xy$; $y(0) = 1$, $y(0.5) = ?$ & $y(1) = ?$

Sol^n Given $\boxed{\frac{dy}{dx} = f(x, y) = xy}$ $x_0 = 0$, $y_0 = 1$, $h = 0.5$

$\Rightarrow y_1 = y_0 + hf(x_0, y_0)$

$\Rightarrow y_1 = 1 + 0.5 * (x_0 y_0)$

$\Rightarrow y_1 = 1 + 0.5 \times 0$

$\Rightarrow \boxed{y_1 = 1}$

$$\Rightarrow y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1)\right]$$

$$\Rightarrow y_1^{(1)} = 1 + \frac{0.5}{2}\left[0 + 0.5 \times 1\right]$$

$$\Rightarrow \boxed{y_1^{(1)} = 1.125}$$

Now,

$$\Rightarrow y_1^{(2)} = 1 + \frac{0.5}{2}\left[0 + 0.5 \times 1.125\right]$$

$$\Rightarrow y_1^{(2)} = 1 + 0.25 \times 0.5 \times 1.125$$

$$\Rightarrow \boxed{y_1^{(2)} = 1.1406}$$

★ Continue till we get the same value as previous one.

Runge Kutta method

$$y_1 = y_0 + k$$

$$k_1 = hf(x_0, y_0), \quad k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right), \quad k_4 = hf\left(x_0 + h, y_0 + \frac{k_3}{2}\right)$$

$$k = \frac{k_1 + 2k_2 + 2k_3 + k_4}{6}$$